SOME NONSTABLE HOMOTOPY GROUPS OF LIE GROUPS

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The main result of [6], stating that S^{4n-1} is not parallelizable except for $n = 1$ and 2, can be reformulated in terms of homotopy groups of the rotation group $SO(4n-1)$ as follows: For $n \geqq 3$, $\pi_{4n-2}(SO(4n-1))$ is not zero; or equivalently, for $n \geqq 3$, $\pi_{4n-2}(SO(4n-2))$ is not zero. (Compare [6], Lemma 2.)

In the present paper, the results of R. Bott [2] on the stable homotopy of the classical groups and the isomorphism $\pi_{2q}(U(q)) \cong Z/q!Z$ are used to derive more precise information on $\pi_{4n-2}(SO(4n-1))$, $\pi_{4n-2}(SO(4n-2))$, and further nonstable homotopy groups of the rotation group $SO(m)$ and the unitary group $U(m)$. Our results also rely essentially on the computations of G. F. Paechter [8].

As seen from the tables below, periodicity persists "for some time" in the nonstable range in the sense that $\pi_{r+m}(SO(m))$ for $r \leqq 1$ and large m depends only on the remainder class of $r + m$ modulo 8. (Periodicity breaks down for low values of m, due to the fact that S^1, S^3, S^7 are parallelizable.) Similarly, for m large enough and $r \leqq 2$, $\pi_{2m+r}(U(m))$ depends only on the parity of r.

$\pi_{2m+r}(U(m))$ is given for $r \leqq 2$ by the following table:

$r \backslash m$	$2k-1$	$2k$
1	0	Z_2
2	$Z_{(2k)!/2}$	$Z_2 + Z_{(2k+1)!}$ for $k > 1$ Z_{12} for $k = 1$

$\pi_{m+r}(SO(m))$ is given by the following table, valid for $s \geqq 1$:

$r \backslash m$	$8s$	$8s+1$	$8s+2$	$8s+3$	$8s+4$	$8s+5$	$8s+6$	$8s+7$
-1	$Z+Z$	Z_2+Z_2	$Z+Z_2$	Z_2	$Z+Z$	Z_2	Z	Z_2
0	$Z_2+Z_2+Z_2$	Z_2+Z_2	Z_4	Z	Z_2+Z_2	Z_2	Z_4	Z
1	$Z_2+Z_2+Z_2$	Z_8	Z	Z_2	Z_2+Z_2	Z_8	Z	Z_2+Z_2
2	$Z_{24}+Z_8$	$Z+Z_2$	Z_{12}	Z_2+Z_2	Z_4+Z_{24d}	$Z+Z_2$	$Z_{12}+Z_2$	Z_2+Z_2
3	$Z+Z_2$	0	Z_2	Z_{8d}	$Z+Z_2$	Z_2	Z_2	Z_8
4	0	Z_2	Z_{8d}	$Z+Z_2$	Z_2	Z_2	Z_8	$Z+Z_2$

In this table d is ambiguously 1 or 2.

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