STRONG CONVERSE OF THE CODING THEOREM FOR SEMICONTINUOUS CHANNELS[1]

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1. Introduction

The purpose of the present paper, which is almost entirely self-contained and requires no prior knowledge of information theory, is to prove the strong converse of the coding theorem for the semicontinuous memoryless channel with any distribution of error whatever (arbitrary noise). The exact result will be stated in Section 4. It will be explained below why it is not possible to reduce our problem by the usual discretization procedure to the discrete memoryless channel, for which the strong converse has been proved. (See Theorem 2 of [1]. Actually this result is stronger than the strong converse; this point will be explained later.) We will also explain below the difference between a weak and strong converse; the two are sometimes confused in the literature.

In [2] and [3] the strong converse of the coding theorem was extended to a certain discrete channel with memory.[2] The proofs rested on the strong converse for the discrete memoryless channel. It is easy to carry over both of these proofs to the analogous semicontinuous channel with memory, provided the strong converse for the semicontinuous memoryless channel has already been proved.

Another proof of the theorem is sketched briefly in the last section. The author is of the opinion that modifications of these proofs will apply to a wide variety of channels.

Professor C. E. Shannon has informed the author that in an unpublished paper he proved the strong converse for the discrete memoryless channel. In [4] he proved the strong converse for a particular continuous channel with additive Gaussian noise.

2. The semicontinuous memoryless channel

We assume, without essential loss of generality, that the alphabet of letters sent or transmitted consists of two symbols, say zero and one. The extension of our results to the case when this alphabet contains any finite number of symbols is trivial.

The channel probability function consists of a pair (because the alphabet of transmitted symbols contains two elements) of probability distribution functions F_0 and F_1. For simplicity of exposition we will take these to be

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[2] *Added in proof*. The author has since proved the strong converse of the coding theorem for what is called in [10] and [3] the general discrete finite-memory channel.