ON THE SOLUTION OF AN IMPLICIT EQUATION

BY

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In this paper a method of solving an implicit equation $g(x_1, \cdots, x_n; y) = 0$ by successive substitutions is given. The customary hypotheses that the function g be differentiable and that $g = 0$ at a given point are replaced by weaker hypotheses.

Appraisals of the remainder error are given, as well as a method of minimizing one of them. Three of the appraisals are valid regardless of miscalculations at earlier stages of the work.

It is also proved that if, in addition, the function g satisfies a Lipschitz condition in a subset of the x_i's, then the solution $y = Y(x_1, \cdots, x_n)$ will also satisfy a Lipschitz condition in the same subset.

In the statements and proofs which follow, unless otherwise specified the index i runs from 1 to n; $(x) \equiv (x_1, \cdots, x_n)$ and $(x; y) \equiv (x_1, \cdots, x_n; y)$. All independent variables and functions mentioned are understood to be real, and the functions single valued.

THEOREM 1. *Let $g(x_1, \cdots, x_n; y) \equiv g(x; y)$ bs a continuous function defined on the closed region $N_1 \subset E^{n+1}$ determined by*

$$(1) \qquad | x_i - a_i | \leqq \alpha_{i1}, \qquad | y - b | \leqq \beta,$$

where α_{i1} and β are positive constants, and let there be constants C and D such that

$$(2) \qquad | g(a; b)| < C\beta,$$

and, if $(x; u)$ and $(x; v)$ are any two distinct points of N_1,

$$(3) \qquad 0 < C \leqq \frac{g(x; u) - g(x; v)}{u - v} \leqq D.$$

Then there exist n positive constants $\alpha_i \leqq \alpha_{i1}$ and a continuous function $Y(x)$ such that, if T is the closed region determined by $| x_i - a_i | \leqq \alpha_i$, the locus of the equation $y = Y(x)$ for $x \in T$ is the same as that of $g(x; y) = 0$ for $(x; y) \in N$, where $N \subset N_1$ is the closed region determined by

$$| x_i - a_i | \leqq \alpha_i, \qquad | y - b | \leqq \beta.$$

We shall prove Theorem 1 simultaneously with Theorem 2.

THEOREM 2. *The constants α_i of Theorem 1 can be chosen subject only to the condition*

$$(4) \qquad | g(x; b) | < C\beta, \qquad | x_i - a_i | \leqq \alpha_i .$$

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