PRINCIPAL QUASIFIBRATIONS AND FIBRE HOMOTOPY EQUIVALENCE OF BUNDLES[1]

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Introduction

Let H be a topological space with a continuous multiplication (an $\mathfrak{H}$-space) which is associative and has a two-sided unit e. In analogy to the case of a topological group we construct a universal principal quasifibration (= q.f.) $\mathfrak{E}_H = (E_H, p_H, B_H, H)$ with fibre H. As an application we get a classification of fibre bundles with respect to fibre homotopy equivalence (see 7.6).

The universal q.f. $\mathfrak{E}_H$ is obtained by iteration of a construction which is described in §2. This construction applies to any q.f. $p:E \to B$ (see §1) in which a (not necessarily associative) $\mathfrak{H}$-space H operates (see Definition 2.2). In a functorial way it embeds such a q.f. into a bigger q.f.

$$\begin{array}{ccc} E & \overset{f}{\subset} & \hat{E} \\ p\downarrow & & \downarrow\hat{p} \\ B & \subset & \hat{B} \end{array}$$

such that the inclusion f is nullhomotopic (see 2.3). In particular there exists always a q.f. whose fibre is homeomorphic with H and is contractible to a point in the total space; we have only to begin with the fibration $H \to P$ which sends all of H into a point P, and in which H operates by right translations.

We speak of a principal q.f. $\mathfrak{E} = (E, p, B, H)$ (see 3.1) if H is associative and $p:E \to B$ is a q.f. in which H operates such that $(yh)h' = y(hh')$ ($y \in E$, $h, h' \in H$). Applying the construction of §2 to a principal q.f. $\mathfrak{E}$ gives a principal q.f. $\hat{\mathfrak{E}}$. Iteration gives a sequence $\mathfrak{E}_{n+1} = \hat{\mathfrak{E}}_n$ of principal q.f.s together with inclusion maps. By taking the limit (in a proper way; see 3.4) of this sequence one obtains a principal q.f. $\mathfrak{E}_\infty = (E_\infty, p_\infty, B_\infty, H)$ which is universal in the sense that all homotopy groups of E_∞ vanish (see 3.5). In particular there exists always a universal principal q.f. with fibre H; as above, one has only to start with a fibration $H \to P$ (= a point).

If $H = G$ is a topological group and we begin our construction with a principal bundle $\mathfrak{E}$ in the sense of Steenrod [6], then $\hat{\mathfrak{E}}$ and $\mathfrak{E}_\infty$ are principal bundles (see 4.1), and our construction coincides (see 4.2) essentially with Milnor's construction in [4].

In §§4–5 universal principal q.f.s are used for a partial homotopy classifi-

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