TOROID TRANSFORMATION GROUPS ON EUCLIDEAN SPACE

BY

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1. Introduction

We deal here with the operation of an r-dimensional toroid T^r on a *cohomology manifold* X^n which resembles euclidean n-space in a sense that is described below. Our main results give a detailed description of the action of T^r on X^n when $n \leqq 2r + 1$ and T^r operates effectively. We prove for example that the fixed point set F is either a point or a line, that all the isotropy subgroups are connected, that T^r has exactly 2^r isotropy subgroups, 2^r fixer subgroups, and r weights (see Section 3 for definitions).

In the case of a general r and n, we prove the inequality $0 \leqq \dim F \leqq \dim X - 2r$, provided that T^r operates almost effectively. Here F is a nonempty cohomology manifold resembling a euclidean space in the sense of our definition. Most of the proofs rely on recursion processes, which are based on the existence of *star circle subgroups*, that is, circle subgroups P of T^r whose fixed point sets are not the fixed point sets of any connected subgroup properly containing P (see Section 3).

In the case of an effective T^r on an HLC euclideanlike X^n with $n \leqq 2r + 1$ (in view of the aforementioned inequality, $n = 2r$ or $2r + 1$), we prove (Section 6) that the space of principal orbits U admits a global cross section and $U = B \times T^r$ where B is a euclideanlike cohomology manifold. The added hypothesis that X is an HLC space has been introduced, in order to allow us to employ Poincaré duality for open cohomology manifolds *over the integers* (see Section 4, Corollary 4.1). The proof of the existence of the global cross section involves a generalization of the fact that any map of euclidean space is homotopic to a constant (see Section 5).

Our results show that for the cases considered the action is closely related to the known linear action. The reader may find it helpful to keep the linear case in mind and interpret the definitions in this light. It can be shown that the action need not be equivalent to a linear action, however. This is not immediate but could be concluded, for example, from making use of some recent results of Bing (to appear in Annals of Mathematics) showing that E^4 is the product of a line and a space which is not a manifold.

2. Generalized manifolds

All of the spaces considered in this paper are locally compact, Hausdorff, and finite-dimensional. The cohomology groups (Alexander-Spanier) with coefficients in a group L are denoted by $H^i(X, L)$ and those with compact

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