PRODUCTS OF GENERALIZED MANIFOLDS

BY
THOMAS R. BRAHANA

1. Introduction

Generalized manifolds were defined by Čech, and are studied in detail in R. L. Wilder's book, *Topology of Manifolds* [5]. It is the purpose of this paper to prove that the product of generalized manifolds is a generalized manifold, subject to restrictions of a dimension theoretic nature. A more general statement may be made: the bundle space of a fibre bundle whose base and fibre are in the class of generalized manifolds referred to is a generalized manifold.

The proof depends upon a formula for determining the local Betti numbers of a point in the product of two locally compact spaces. This formula, analogous to the Künneth formula for determining the homology of product spaces, is given in Theorem 1.

2. Preliminaries

Generalized manifolds are defined by conditions on the local homology of the space. In particular, the local Betti numbers are used, and defined as follows (see [5], p. 191).

Given a space S and a point $x \in S$, let $\{P_\alpha\}$ be a basis for the open sets containing x, and for each α let $\{Q_{\alpha\beta}\}$ be a basis for the open sets of S containing x and contained in P_α. The symbol $Z_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})$ represents the vector space of q-dimensional Čech cycles on S mod $(S - P_\alpha)$, with coefficients in a field. The symbol $B_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})$ denotes the subspace of the above consisting of those cycles which bound on S mod $(S - Q_{\alpha\beta})$.

The indices $\{\alpha\}$ of the $\{P_\alpha\}$ are ordered by inclusion, i.e. $\alpha_1 < \alpha_2$ if and only if $P_{\alpha_1} \supset P_{\alpha_2}$. In a similar manner the indices $\{\alpha\beta\}$ are ordered by the relation, $\alpha_1 \beta_1 < \alpha_2 \beta_2$ if and only if $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$.

The generalized limit

$$\lim_{\alpha\beta} \dim\, [Z_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})/B_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})],$$

which is induced by the order relation among the $\{\alpha\beta\}$, exists (or may consistently be called infinite), since

$$\dim\, [Z_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})/B_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})]$$

is nonincreasing for $\alpha\beta < \alpha_1 \beta_1$, i.e. $Q_{\alpha\beta} \supset Q_{\alpha_1\beta_1}$. The double limit

$$\lim_{\alpha} \lim_{\alpha\beta} \dim\, [Z_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})/B_q(x:S, S - P_\alpha; S, S - Q_{\alpha\beta})]$$

Received April 19, 1957