ALGEBRAIC CLOSURE OF FIELDS AND RINGS OF FUNCTIONS

Dedicated to L. J. Mordell in gratitude and friendship on his seventieth birthday, January 28, 1958

BY

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The class of rings of functions that is going to be the object of our discussion may be described as follows: There are given firstly a [commutative] field F, the field of values of the ring of functions; secondly a set D of elements [called points], the domain of the ring of functions; and thirdly and mainly a ring R of single-valued functions, defined on D with values in F. [Addition and multiplication of functions in R are defined in the natural fashion:

$$(f + g)(x) = f(x) + g(x), \qquad (fg)(x) = f(x)g(x)$$

for x in D and f, g in R.] These rings will always be subject to the following requirements:

R contains all the constants;

if x and y are different points in D, then there exists a function f in R such that $f(x) \neq f(y)$.

All these rings are commutative and contain a ring unit 1, namely the constant 1. The requirement that all constants are present in R is not quite as harmless as it appears. The field of constants which is naturally isomorphic with the field F of values shall be denoted by C. The requirement on the other hand that there exists to any pair of different points in D a function in R which takes different values on these points does not constitute a loss of generality, since we would form otherwise the classes of points in D on which all functions in R take the same value, and since we could consider these classes as the "points".

With such a configuration $[F, D, R]$ we connect two topological spaces.

The space of maximal ideals

We denote by $T = T(R)$ the totality of maximal ideals in R. If p is a point in T and S is a subset of T, then p is said to belong to the closure $\bar{S}$ of S if, and only if,

$$S^* = \bigcap_{s \in S} s \leqq p.$$

It is well known that T with the topology just described is a compact T_1-space [so that in particular every point is a closed set and every covering of T with open sets contains a finite covering of T]; see Jacobson [1] or Samuel [1; pp.

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