GENERALIZED INCIDENCE MATRICES OVER GROUP ALGEBRAS

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1. Introduction

In previous papers [3, 4] the author has investigated certain matrix equations which must hold if a (v, k, λ) configuration is to possess collineations. These equations involved matrices with rational entries, and the Hasse-Minkowski theory of rational congruence was applied to give numerical conditions restricting the possible collineations of a (v, k, λ) configuration. The author has found that these rational matrix equations are in fact derivable from more "general" equations involving matrices over a group algebra, and that these latter equations yield at least one result which is not deductible by the rational congruence methods of the earlier papers; if π is a projective plane of order $n \equiv 2 \pmod 4$, $n \neq 2$, then π possesses no collineations of even order. However, the general problems presented by the group algebra equations appear to be difficult of solution.

2. Group algebra matrices

We shall rely heavily on [4] for background material, but a brief review of some basic topics will be given. Let v, k, λ be integers satisfying $v > k > \lambda > 0$ and $\lambda(v - 1) = k(k - 1)$, and let π be a collection of v *points* and v *lines*, together with an *incidence relation* satisfying: (i) each point (line) is on k lines (contains k points), and (ii) each pair of distinct points (lines) is on λ common lines (contains λ common points). Then π is a (v, k, λ) *configuration*, and we define the *order* n of π by $n = k - \lambda$; if $\lambda = 1$, then π is a *projective plane* of order n. A *collineation* of π is a one-to-one mapping of points onto points and lines onto lines which preserves incidence. A collineation group $\mathfrak{G}$ of π is called *standard* if every non-identity element of $\mathfrak{G}$ fixes the same set of points and lines; any collineation group of prime order is standard.

Suppose π is a (v, k, λ) configuration and $\mathfrak{G}$ is a collineation group of π, where $\mathfrak{G}$ has order m. From Theorem 2.3 of [4] we know that the number of transitive classes of points equals the number of transitive classes of lines (X and Y are in the same transitive class if and only if $X = Yb$ for some b in $\mathfrak{G}$). We number the transitive classes of points (lines) $1, 2, \cdots, w$, and let P_i (J_i) be an arbitrary but fixed point (line) in the i^{th} transitive class of points (lines). Let $\mathfrak{P}_i$ ($\mathfrak{J}_i$) be the subgroup of $\mathfrak{G}$ which fixes P_i (J_i), and let $\mathfrak{P}_i$ ($\mathfrak{J}_i$) have order r_i (s_i). Let D_{ij} be the set of all x in $\mathfrak{G}$ such that $P_i x$ is on J_j.

Let F be a field whose characteristic does not divide any of the numbers r_i or s_i; if $\mathfrak{K}$ is a group, we denote by $\mathfrak{A}(\mathfrak{K})$ the group algebra of $\mathfrak{K}$ over F.

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