SOME DUALITY THEOREMS[1]

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1. Basic notions

Certain concepts used in the theory of group representations apply equally to matrix-valued functions defined on a set S. For instance, if $f: S \to M_1$ and $g: S \to M_2$ where M_i is the total matrix algebra over some field ($i = 1, 2$), then the Kronecker product $f \times g$ is defined just as for representations. Similarly, the concept of irreducibility also carries over. f will be called irreducible if f maps S onto an irreducible set of matrices.[2]

Suppose G is a compact topological group, and R_1, R_2 representations of G. According to a basic theorem, the Kronecker product $R_1 \times R_2$ "decomposes" into irreducible components. More precisely, there exist irreducible representations $P_1, P_2, \cdots, P_k$ of G, positive integers $m_1, m_2, \cdots, m_k$, and a nonsingular matrix A, such that

$$(1) \qquad R_1 \times R_2 = A \begin{pmatrix} \left.\begin{matrix} P_1 & & & \\ & P_1 & & \\ & & \ddots & \\ & & & P_1 \end{matrix}\right\} m_1 \text{ times} & & & \\ & \left.\begin{matrix} P_2 & & & \\ & P_2 & & \\ & & \ddots & \\ & & & P_2 \end{matrix}\right\} m_2 \text{ times} & & \\ & & \ddots & \\ & & & \begin{matrix} P_k & & & \\ & P_k & & \\ & & \ddots & \\ & & & P_k \end{matrix} \end{pmatrix} A^{-1},$$

where the big matrix above is to be completed with zero matrices. We shall denote this matrix by $\Delta_{i=1}^{k} m_i P_i$.

Systems of matrix-valued functions which satisfy algebraic relations of the type (1) will be of interest. For this purpose we make the following definition.

Received October 1, 1956.

[1] This paper is part of a doctoral dissertation written at the University of Pennsylvania under the sponsorship of Professor N. J. Fine.

[2] For a general discussion of matrix-valued functions on a set S, see [1], Ch. VI where they are discussed under the name of "S-modules".