SINGULAR HYPERSURFACES IN GENERAL RELATIVITY

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1. Introduction

The general theory of relativity is concerned with a four-dimensional Riemannian space, space-time, with a metric

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu. \tag{1.1}$$

The components of the metric tensor $g_{\mu\nu}$ represent the gravitational fields in the coordinate system x^μ. They are related to the matter present by means of the field equations

$$R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R = -kc^2T^{\mu\nu}, \tag{1.2}$$

where $R_{\mu\nu}$ is the Ricci tensor of the Riemannian space whose metric is given by equations (1.1), R is the scalar curvature of this space,

$$k = 8\pi G/c^2 \tag{1.3}$$

with G being Newton's constant of gravitation, c the special theory of relativity velocity of light, and $T^{\mu\nu}$ the stress energy tensor of the matter present. In writing equations (1.2) we have chosen the dimensions of ds to be those of time.

It is a consequence of equations (1.2) that

$$T^{\mu\nu}{}_{;\nu} = 0, \tag{1.4}$$

where the semicolon represents the covariant derivative with respect to the tensor $g_{\mu\nu}$ and the summation convention has been used. Equations (1.4) are the equations of motion of the matter present and restrict the specification of the tensor $T^{\mu\nu}$ in equations (1.2).

These equations have been discussed in great detail [1], [2] for the case where the space-time has plane symmetry and the matter present is a perfect fluid with stress energy tensor

$$T^{\mu\nu} = \sigma u^\mu u^\nu - g^{\mu\nu}p/c^2, \tag{1.5}$$

where u^μ is the four-dimensional velocity vector of the fluid and satisfies

$$g_{\mu\nu}\, u^\mu u^\nu = u^\mu u_\mu = 1, \tag{1.6}$$

p is the pressure, and

$$\sigma = \rho\left(1 + \frac{\varepsilon}{c^2} + \frac{p}{\rho c^2}\right), \tag{1.7}$$

where ρ is the density as measured by an observer at rest with respect to the fluid, and ε is the specific internal energy of the fluid measured similarly.

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