MARKOFF PROCESSES AND POTENTIALS II

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The first five sections of this installment treat a situation which is related to the one considered in the first installment, pages 44 to 93 of this volume, just as the potential theory of the Laplacian in a region is related to the theory of the Newtonian potential in the whole of Euclidean space. The following section shows the relative theory to be in a sense complete; and the last section sketches a slight extension—or rather another interpretation—of the main theorems.

The numbering continues that of the first installment. References such as [1] are to the list at the end of the first installment.

10. Terminal times

The simple terminal time S, which serves only to produce convergence, will now be replaced by one defined in terms of a positive function a and a set A. Loosely speaking, the new terminal time is the moment a wandering particle is destroyed, if there is probability $a(r)\,d\tau$ that the particle, having reached the point r safely, is destroyed in the subsequent time interval $d\tau$ and if in addition the particle is sure to be destroyed the instant it touches A.

To be precise, let a be a positive function measurable over the field $\mathfrak{A}$ and let A be a nearly analytic set. Given a process X and a positive random variable Z_X, independent of the process and having the density function $e^{-\sigma}$ for positive σ, define $R_X(\omega)$ to be the infimum of the strictly positive τ for which at least one of the statements

$$(10.1) \qquad X(\tau, \omega) \in A, \qquad \int_0^\tau a\big(X(\sigma, \omega)\big)\, d\sigma \geqq Z_X(\omega),$$

is true, with the understanding that $R_X(\omega)$ is infinite if there are no such τ. We shall say that R_X is the terminal time assigned to X by a and A, with Z_X as auxiliary variable.

Let T be a stopping time for X, with $\mathcal{E}$ as auxiliary field, and suppose that X, Z_X, $\mathcal{E}$ are independent and that Ω', the set where T is less than R_X, has strictly positive probability. Take Y to be the process

$$Y(\tau, \omega) = X\big(\tau + T(\omega), \omega\big), \qquad \tau \geqq 0,\ \omega \in \Omega',$$

defined over the probability field Ω', and take R_Y to be the restriction of $R_X - T$ to Ω'. Straightforward calculation shows that R_Y is in fact that terminal time assigned to Y by a and A, the auxiliary variable being the

Received January 10, 1957.

[1] The preparation of this paper was supported by the United States Air Force, through the Office of Scientific Research of the Air Research and Development Command.