CLASSES OF FINITE GROUPS AND THEIR PROPERTIES

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Of the various properties that a class Θ of finite groups may or may not have, those of interest to us in our present investigation can be described roughly as follows:

1. The formal or inheritance properties: Subgroups, homomorphic images, and direct products of groups in Θ may or may not belong to Θ; and somewhat less superficial is the question whether a product of normal Θ-subgroups is itself a Θ-group; see e.g. Specht [1; §1.4.4. etc.].

2. The material properties: These are concerned with the structure of the minimal normal subgroups of homomorphic images of Θ-groups and with the structure of the automorphism groups induced by homomorphic images of Θ-groups in their minimal normal subgroups. They are furthermore concerned with the situation of maximal subgroups in Θ-groups. In particular one wants to derive from such properties criteria for a group to be a Θ-group, criteria that will lead to theorems asserting that a group G is a Θ-group if, and only if, $G/\Phi(G)$ is a Θ-group.

3. Somewhat in between **1** and **2** are questions of the following type: Is a group G a Θ-group if, and only if, every n-tuplet of elements in G, for n a fixed integer, generates a Θ-group? Is furthermore G a Θ-group, if there exists a normal subgroup N of G such that G/N and every $\{N, x_1, \cdots, x_n\}$ for x_i in G and n a fixed integer is a Θ-group?

One might try to undertake such an investigation completely in abstracto, attempting to derive relations between such general properties of a class Θ of finite groups; and some few results of such generality will be found in the present investigation. But we have been concerned here mainly with more concrete questions; and the starting point of our investigation was the observation that such properties as supersolubility, nilpotence, dispersion, existence of Sylow towers, nilpotence of the commutator subgroup are highly complex and may be reduced to more elementary properties in the sense that they are equivalent to certain concatenations of these elementary properties—that different sets of such elementary properties may characterize one and the same complex property, leads to particularly intriguing problems. We have considered here just two types of elementary properties. The first one is

Σ-*closure*: If Σ is a set of primes and if the set of elements in the group G whose orders are divisible by primes in Σ only is actually a subgroup of G, then G is termed Σ-closed.

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