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ERRATUM: CUBIC FOURFOLDS AND SPACES OF RATIONAL CURVES

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This is an erratum to the paper [1]. In the proof of Proposition 3.3, we refer to the paper [3], whose main result is wrong. A counter example to the main result of [3] is the ring map $\mathbf{C}[u^2, v^2, uv] \to \mathbf{C}[u, v]$. But in the proof of Proposition 3.3, we only need the following statement: Suppose A is a domain, smooth over a field k; suppose $A \to B$ is a finite ring map, and assume that $A_f \to B_f$ is finite etale for some nonzero $f \in A$. Then there is a trace map $\mathrm{Tr} : \Omega^p_{B/k} \to \Omega^p_{A/k}$ which extends the obvious map

$$\Omega^p_{B_f/k} = \Omega^p_{A_f/k} \otimes_{A_f} B_f \longrightarrow \Omega^p_{A_f/k}$$

defined using the trace on functions. We sketch a proof of this.

Namely, let B' be the normalization of B in the ring B_f. Since B maps to B' and $B_f = B'_f$, we may replace B by B'. Thus, we may in addition assume that B is a normal ring of finite type over k. In this case, B' is regular away from codimension 2, and hence a local complete intersection ring over k away from codimension 2. In particular, we see that $A \to B$ is a finite locally complete intersection morphism away from codimension 2, and hence [2] applies away from codimension 2 on $\mathrm{Spec}(A)$. However, since $\Omega^i_{A/k}$ are finite locally free, the sections we obtain extend to all of $\mathrm{Spec}(A)$ by Hartog's theorem.

Finally, in the Introduction of the paper, we state that the moduli space M_e of degree e rational curves in X is irreducible for any smooth cubic fourfold X in $\mathbf{P}^5_{\mathbf{C}}$. This may well be true, but we only prove this in the paper in the case X is general (Proposition 2.4). As far as we know the question of irreducibility of M_e for any nonsingular X is still open.

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