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A CUBIC DIRAC OPERATOR AND THE EMERGENCE OF EULER NUMBER MULTIPLETS OF REPRESENTATIONS FOR EQUAL RANK SUBGROUPS

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Contents

0. Introduction

0.1. The results in this paper arise from two distinct origins. The first, and more recent result, grew out of the joint paper [GKRS]. The second is explained in §0.28.

The paper [GKRS] offered, in very general terms, a mathematical explanation of an interesting phenomenon discovered, empirically, by the physicists Ramond and Pengpan. Presumably motivated by a possible connection with M-theory, they found that there was an infinite set $\mathcal{S}$ of irreducible representations of $\mathrm{Spin}(9,\mathbb{R})$ which partitioned into triplets

$$\mathcal{S} = \bigcup_{i\in I} \{\sigma_1^i, \sigma_2^i, \sigma_3^i\}, \tag{0.2}$$

where the representations in each triplet are related to each other in remarkable ways. For example, the infinitesimal character value of the Casimir operator is constant on the triplet, and there are a number of other infinitesimal character relations on the triplet involving more of the generators of $\mathfrak{Z}(\mathfrak{r})$. Here $\mathfrak{r} = \mathrm{Lie}\ \mathrm{Spin}(9)$ and $\mathfrak{Z}(\mathfrak{r})$ is the center of the enveloping algebra $U(\mathfrak{r})$ of $\mathfrak{r}$. Also one has for each $i \in I$,

$$\dim \sigma_1^i + \dim \sigma_2^i = \dim \sigma_3^i. \tag{0.3}$$

The simplest triplet $\{\sigma_1^1, \sigma_2^1, \sigma_3^1\}$ arises from the irreducible 16-dimensional (spin)

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