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FACTORIZATION THROUGH MATRIX SPACES FOR FINITE RANK OPERATORS BETWEEN C^*-ALGEBRAS

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0. Introduction. In this paper we consider factorizations of finite rank operators through finite-dimensional C^*-algebras. We are interested in factorization norms involving either the completely bounded norm $\| \ \|_{\mathrm{cb}}$ or Haagerup's decomposable norm $\| \ \|_{\mathrm{dec}}$ (see [11]). Let us denote by M_n the C^*-algebra of all $n \times n$ matrices with complex entries. Let A and B be two C^*-algebras, and let us consider a finite rank bounded operator $u\colon A \to B$. Then for n large enough, say $n \geq rk(u)$, we may write factorizations of the form $u = \beta\alpha$, for some bounded operators

$$A \xrightarrow{\alpha} M_n \xrightarrow{\beta} B. \tag{0.1}$$

Our main result (Theorem 2.1) says that for any $\varepsilon > 0$, one can find α and β as above such that $\|\alpha\|_{\mathrm{cb}}\|\beta\|_{\mathrm{dec}} \leq (1+\varepsilon)\|u\|_{\mathrm{dec}}$. If u is completely positive, then $\|u\|_{\mathrm{dec}} = \|u\|$; hence in that case, we obtain that $\|u\| = \inf\{\|\alpha\|_{\mathrm{cb}}\|\beta\|_{\mathrm{dec}}\}$, where the infimum runs over all factorizations as above. This new result that finite rank, completely positive maps factor through matrix algebras gives some explanation of the phenomenon behind the classical result of Choi-Effros-Kirchberg [4], [16] characterizing nuclear C^*-algebras either by the completely positive approximation property or equivalently by the *approximate matriciality* of the algebra.

For a finite rank operator $u\colon A \to B$ between C^*-algebras, let us now introduce $\gamma(u) = \inf\{\|\alpha\|_{\mathrm{cb}}\|\beta\|_{\mathrm{cb}}\}$, where the infimum runs over all $n \geq 1$ and all factorizations of u of the form (0.1). We obviously have $\|u\|_{\mathrm{cb}} \leq \gamma(u)$. In Section 3 we consider the natural problem of whether the converse inequality holds, that is, $\|u\|_{\mathrm{cb}} = \gamma(u)$. We show that this holds if B has the weak expectation property (as defined in [18]). In the case when B is a von Neumann algebra, we obtain the following characterization. The equality $\|u\|_{\mathrm{cb}} = \gamma(u)$ holds for all A and all u if and only if B is injective. This result shows in particular that the two norms $\gamma(\ )$ and $\| \ \|_{\mathrm{cb}}$ may be different. Thus the decomposable norm $\| \ \|_{\mathrm{dec}}$ behaves better than the completely bounded one when dealing with factorization through matrix algebras.

Our factorization results have several applications to the theory of operator spaces. First, we give a new proof of the recent theorem by Effros-Junge-Ruan [6], which asserts that given any von Neumann algebra R, the predual operator space R_* is locally reflexive in the sense of [5], [7]. Second, we show that given any two C^*-algebras A and B, the Banach spaces of completely integral maps and completely 1-summing

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