Vol. 100, No. 1 DUKE MATHEMATICAL JOURNAL © 1999

THE FAKE MONSTER FORMAL GROUP

RICHARD E. BORCHERDS

Contents

1. Introduction. The main result of this paper is the construction of "good" integral forms for the universal enveloping algebras of the fake monster Lie algebra and the Virasoro algebra. As an application, we construct formal group laws over the integers for these Lie algebras. We also prove a form of the no-ghost theorem over the integers and use this to verify an assumption used in the proof of the modular moonshine conjectures.

Over the integers, the universal enveloping algebra of a Lie algebra is not very well behaved, and it is necessary to use a better integral form of the universal enveloping algebra over the rational numbers. The correct notion of "good" integral form was found by Kostant [K]. He found that the good integral forms are the ones with a structural base (see Definition 2.3), and he showed that universal enveloping algebras of finite-dimensional semisimple Lie algebras have a structural base. The existence of a structural base implies that the dual algebra of the underlying coalgebra is a ring of formal power series. Because this ring can be thought of as a sort of "coordinate ring" of some sort of formal group, this condition can be thought of as saying that the formal group is smooth and connected and comes from a "formal group law."

The main point of this paper is to find such integral forms for the universal enveloping algebras of certain infinite-dimensional Lie algebras. We recall that the universal enveloping algebra of a Lie algebra has a natural structure of a cocommutative Hopf algebra, so we need some theorems to tell us when a Hopf algebra has a structural base. In Section 2 we prove that a Hopf algebra H has a structural base, provided

Received 1 June 1998. Revision received 29 December 1998.

1991 *Mathematics Subject Classification*. Primary 17B67; Secondary 17B68, 17B35, 17B69, 14L05.

Author supported by a Royal Society professorship and by National Science Foundation grant number DMS-9401186.