Vol. 100, No. 1 DUKE MATHEMATICAL JOURNAL

MICROLOCAL ANALYTIC SMOOTHING EFFECT FOR THE SCHRÖDINGER EQUATION

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0. Introduction and main results. The purpose of this work is to study the microlocal analytic smoothness of solutions of the initial value problem for the linear Schrödinger equation with variable coefficients. The aim is to relate the behavior at infinity of the initial data with the microlocal analytic smoothness; this phenomenon is known as the microlocal smoothing effect. The results presented here are extensions to the analytic case of those of Craig-Kappeler-Strauss [CKS], [C], which concern the C^∞ case. However, our method of proof, which relies on Sjöstrand theory [Sj], is entirely different from that of [CKS]. This question has also been investigated in recent years in the papers of Shananin [Sh], Kapitanski-Safarov [KS], and Wunsch [W]. Related results have also been obtained by Doi [D1], [D2], and we refer to the paper [CKS] for more references on the subject.

Let us describe our main result. Let $P = P(y, D_y)$, a second-order differential operator in R^n,

$$P = \sum_{j,k=1}^{n} a_{jk}(y) D_j D_k + \sum_{j=1}^{n} a_j(y) D_j + a(y), \quad D_j = \frac{1}{i}\frac{\partial}{\partial y_j}. \tag{0.1}$$

We assume that P has analytic coefficients in $\mathbb{R}^n$ and a real principal symbol p. We make the following assumptions.

(0.2) There exists $\nu > 0$ such that $p(y,\eta) \geq \nu|\eta|^2, \quad \forall (y,\eta) \in T^*\mathbb{R}^n$.

One can find constants $C_0 \geq 1, R_0 \geq 1, K_0 \geq 1$, and $\sigma_0 \in]0,1[$ such that, for all $y \in \mathbb{R}^n, |y| > R_0$, and $\alpha \in \mathbb{N}^n$,

$$\sum_{j,k=1}^{n} \left| D_y^\alpha \big(a_{jk}(y) - \delta_{jk}\big) \right| \leq C_0 K_0^{|\alpha|} \alpha! |y|^{-(1+\sigma_0+|\alpha|)}, \tag{0.3}$$

where δ_{jk} is the Kronecker symbol and

$$\sum_{j=1}^{n} |D_y^\alpha a_j(y)| + |D_y^\alpha a(y)| \leq C_0 K_0^{|\alpha|} \alpha! |y|^{-(1+\sigma_0+|\alpha|)}. \tag{0.4}$$

Let $\rho_0 = (y_0, \eta_0) \in T^*\mathbb{R}^n \setminus \{0\}$. We consider the bicharacteristic of p passing through ρ_0. It is given by the system of differential equations

Received 19 May 1998. Revision received 8 October 1998.
1991 *Mathematics Subject Classification*. Primary 35J10; Secondary 35A20, 35A27, 35Q55.