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EQUIDISTRIBUTION OF HOLONOMY ABOUT CLOSED GEODESICS

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To Professor Shingo Murakami on the occasion of his seventieth birthday

Contents

Introduction. There is a well-known analogy between closed geodesics (especially primitive ones) on Riemannian manifolds X of dimension d of negative curvature and primes. For example, for $x > 0$, let

$$E(x) = \{C; C \text{ is a closed geodesic on } X, \ell(C) \le x\}. \tag{1}$$

Here the number $\ell(C)$ is the length of C. Also set $E_P(x)$ to be the set defined in the same way but restricting C to be primitive. If X is compact, then Margulis [Mar] showed that there is a positive constant $h = h(X)$ such that the analogue of the prime-number theorem holds:

$$|E(x)| \sim |E_P(x)| \sim \frac{e^{hx}}{hx} \quad \text{as } x \to \infty. \tag{2}$$

Analogues of the Chebotarev equidistribution theorem, which can be viewed as a density theorem for conjugacy classes determined by Frobenius substitutions of a

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