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AN ALGEBRAIC CHARACTERIZATION OF THE AFFINE CANONICAL BASIS

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0. Introduction. The canonical basis of the quantized universal enveloping algebra associated to a simple finite-dimensional Lie algebra was introduced by Lusztig in [L2] via an elementary algebraic definition. The definition was characterized by three main components: the basis was integral, was bar-invariant, and spanned a certain $\mathbf{Z}[q^{-1}]$-lattice $\mathscr{L}$ with a specific image in the quotient $\mathscr{L}/q^{-1}\mathscr{L}$. This algebraic definition does not work for quantized universal enveloping algebras of arbitrary Kac-Moody algebras. The difficulty in constructing a basis for $\mathscr{L}$ arises from the need to define suitable analogues of imaginary root vectors. The definition of the canonical basis for arbitrary type was subsequently made using topological methods [L4]. In [K], Kashiwara gave a suitable algebraic definition of the lattices $\mathscr{L}$ and $\mathscr{L}/q^{-1}\mathscr{L}$, making use of a remarkable symmetric bilinear form on the algebra (introduced by Drinfeld), which led to an inductive construction of the global crystal basis. It was later shown in [GL] that the two concepts—the global crystal basis (algebraic) and the canonical basis (topological)–coincide.

In this paper we synthesize the two aforementioned techniques and construct a crystal basis for the quantized universal enveloping algebra of (untwisted) affine type. We then give an elementary algebraic characterization of the canonical basis analogous to the characterization given in the finite-type case [L2]. A remarkable feature of our construction is that the part of the crystal basis corresponding to the imaginary root spaces is given by Schur functions in the Heisenberg generators. We were motivated to consider the Schur functions for the following reasons. The imaginary root vectors were constructed in [CP], where it was shown that they could be defined by a certain functional equation in terms of the Heisenberg generators. After a suitable renormalization, this equation is the same as the equation that expresses the complete symmetric functions in terms of the power sums. In Section 4 of this paper, we show that the imaginary root vectors generate a polynomial algebra over $\mathbf{Z}[q,q^{-1}]$, are grouplike with respect to the comultiplication, and are quasi-orthonormal with respect to the Drinfeld form on the algebra. It is well known (see [M]) that the complete symmetric functions are also grouplike and orthonormal with respect to the standard Hopf algebra structure and inner product on the ring of symmetric functions,

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