

# HÖLDER REGULARITY AND DIMENSION BOUNDS FOR RANDOM CURVES

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*To the memory of Roland L. Dobrushin*

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## 1. Introduction

*1.1. General framework.* We consider here curves in  $\mathbb{R}^d$  that are shaped on many scales in a manner found in various critical models (see Figure 1). The framework for the discussion is random systems, where the random object is expressed as a closed collection of polygonal curves of a small step size  $\delta$ . Our main results are general

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