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NONVANISHING MODULO ℓ OF FOURIER COEFFICIENTS OF HALF-INTEGRAL WEIGHT MODULAR FORMS

JAN HENDRIK BRUINIER

1. Introduction. Let k be an integer and N be a positive integer divisible by 4. If ℓ is a prime, denote by v_ℓ a continuation of the usual ℓ-adic valuation on $\mathbb{Q}$ to a fixed algebraic closure. Let f be a modular form of weight $k+1/2$ with respect to $\Gamma_0(N)$ and Nebentypus character χ which has integral algebraic Fourier coefficients $a(n)$, and put $v_\ell(f) = \inf_n v_\ell(a(n))$. Suppose that f is a common eigenform of all Hecke operators $T(p^2)$ with corresponding eigenvalues λ_p.

In a recent paper, Ono and Skinner (under the additional assumption that f is "good") proved the following theorem [OS]: For all but finitely many primes ℓ, there exist infinitely many square-free integers d for which $v_\ell(a(d)) = 0$. Their proof uses the theory of ℓ-adic Galois representations. Similar results were obtained by Jochnowitz in [J] by developing a theory of half-integral weight modular forms modulo ℓ analogous to the integral weight theory due to Serre, Swinnerton-Dyer, and Katz.

Results of this type can be viewed as mod ℓ versions of a well-known theorem of Vignéras about the nonvanishing of Fourier coefficients of half-integral weight modular forms (see [V]). A new proof for this was given by the author (see [B]).

In the present paper, we extend the method introduced in [B] to the modulo ℓ situation and thereby obtain a new approach to the above stated theorem and certain generalizations.

We use an application of the q-expansion principle of arithmetic algebraic geometry (Lemma 1) and exploit the properties of various well-known operators defined on modular forms to infer our first result (Theorem 1). Roughly speaking, it states that if for a given prime p and a given $\varepsilon \in \{\pm 1\}$, all Fourier coefficients $a(n)$ with $(\frac{n}{p}) = \varepsilon$ vanish modulo ℓ, then the Hecke eigenvalue λ_p satisfies a certain congruence modulo ℓ.

Under the (obviously necessary) assumption that f is not a linear combination of elementary theta series of weight 1/2 or 3/2, one can deduce several nonvanishing theorems. For instance, in Theorem 4 we show that there exists a finite set $A_N(f)$ of primes that has an explicit description in terms of the eigenvalues λ_p with the following property: For every prime ℓ with $(\ell, N) = 1$, $v_\ell(f) = 0$, and $\ell \notin A_N(f)$, there are infinitely many square-free d such that $v_\ell(a(d)) = 0$. Note that we do not need the notion of a "good" modular form. Theorems 2 and 3 contain certain

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