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FUNDAMENTAL SOLUTIONS FOR THE TRICOMI OPERATOR

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1. Introduction. The second-order equation in two independent variables x and y,

$$\mathcal{T}u = yu_{xx} + u_{yy} = 0, \tag{1.1}$$

known as the Tricomi equation, is a classical example of a partial differential equation of mixed type. The equation is *elliptic* in the half-plane $y > 0$, *parabolic* along the x-axis, and *hyperbolic* in the half-plane $y < 0$.

Our aim is to determine *fundamental solutions* for the Tricomi equation (1.1) with pole at a variable point $(a, 0)$ on the x-axis. These are solutions of the equation

$$\mathcal{T}u = \delta(x-a, y), \tag{1.2}$$

where $\delta(x-a, y)$ is the Dirac measure concentrated at $(a, 0)$. Since the Tricomi equation changes type in any neighborhood of $(a, 0)$, we also analyze the influence that both the elliptic and hyperbolic parts have on the fundamental solutions. In view of the invariance of the Tricomi equation by translations along the x-axis, it suffices to determine fundamental solutions with pole at the origin.

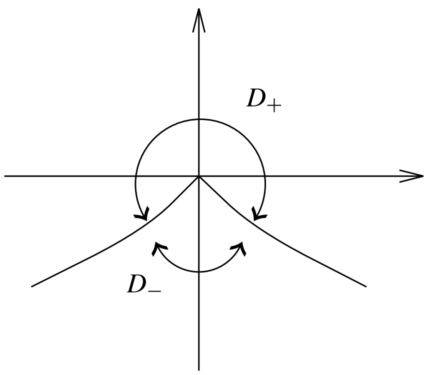

FIGURE 1

Let D_+ be the region in the (x, y) plane defined by

$$D_+ = \left\{(x, y) \in \mathbb{R}^2 : 9x^2 + 4y^3 > 0\right\},$$

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