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p-ADIC PERIODS, p-ADIC L-FUNCTIONS, AND THE p-ADIC UNIFORMIZATION OF SHIMURA CURVES

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CONTENTS

1. Introduction. Let $E/\mathbb{Q}$ be a modular elliptic curve of conductor N, and let p be a prime of split multiplicative reduction for E. Write $\mathbb{C}_p$ for a fixed completion of an algebraic closure of $\mathbb{Q}_p$. Tate's theory of p-adic uniformization of elliptic curves yields a rigid-analytic, $\mathrm{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$-equivariant uniformization of the $\mathbb{C}_p$-points of E:

$$0 \to q^{\mathbb{Z}} \to \mathbb{C}_p^{\times} \xrightarrow{\Phi_{\mathrm{Tate}}} E(\mathbb{C}_p) \to 0, \tag{1}$$

where $q \in p\mathbb{Z}_p$ is the p-adic period of E.

Mazur, Tate, and Teitelbaum conjectured in [MTT] that the cyclotomic p-adic L-function of $E/\mathbb{Q}$ vanishes at the central point to order one greater than that of its classical counterpart. Furthermore, they proposed a formula for the leading coefficient of such a p-adic L-function. In the special case where the analytic rank of $E(\mathbb{Q})$ is zero, they predicted that the ratio of the special value of the first derivative of the cyclotomic p-adic L-function and the algebraic part of the special value of the complex L-function of $E/\mathbb{Q}$ is equal to the quantity

$$\frac{\log_p(q)}{\mathrm{ord}_p(q)}$$

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