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L^2-BOUNDEDNESS OF THE CAUCHY INTEGRAL OPERATOR FOR CONTINUOUS MEASURES

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1. Introduction. Let μ be a continuous (i.e., without atoms) positive Radon measure on the complex plane. The truncated Cauchy integral of a compactly supported function f in $L^p(\mu)$, $1 \leq p \leq +\infty$, is defined by

$$\mathscr{C}_\varepsilon f(z) = \int_{|\xi - z| > \varepsilon} \frac{f(\xi)}{\xi - z}\, d\mu(\xi), \qquad z \in \mathbb{C},\ \varepsilon > 0.$$

In this paper, we consider the problem of describing in geometric terms those measures μ for which

$$\int |\mathscr{C}_\varepsilon f|^2 \, d\mu \leq C \int |f|^2 \, d\mu, \tag{1}$$

for all (compactly supported) functions $f \in L^2(\mu)$ and some constant C independent of $\varepsilon > 0$. If (1) holds, then we say, following David and Semmes [DS2, pp. 7–8], that the Cauchy integral is bounded on $L^2(\mu)$.

A special instance to which classical methods apply occurs when μ satisfies the doubling condition

$$\mu(2\Delta) \leq C\mu(\Delta),$$

for all discs Δ centered at some point of $\operatorname{spt}(\mu)$, where 2Δ is the disc concentric with Δ of double radius. In this case, standard Calderón-Zygmund theory shows that (1) is equivalent to

$$\int \left|\mathscr{C}^* f\right|^2 d\mu \leq C \int |f|^2 \, d\mu, \tag{2}$$

where

$$\mathscr{C}^* f(z) = \sup_{\varepsilon > 0} |\mathscr{C}_\varepsilon f(z)|.$$

If, moreover, one can find a dense subset of $L^2(\mu)$ for which

$$\mathscr{C} f(z) = \lim_{\varepsilon \to 0} \mathscr{C}_\varepsilon f(z) \tag{3}$$

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