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QUANTUM GALOIS THEORY FOR FINITE GROUPS

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Dong and Mason [DM1] initiated a systematic search for a vertex operator algebra with a finite automorphism group, which is referred to as the "operator content of orbifold models" by physicists (see [DVVV]). The purpose of this paper is to extend one of their main results. We assume that the reader is familiar with the vertex operator algebras (VOAs); see [B], [FLM].

Throughout this paper, V denotes a simple VOA, G is a finite automorphism group of V, $\mathbf{C}$ denotes the complex number field, and $\mathbf{Z}$ denotes rational integers. Let H be a subgroup of G, and let $\mathrm{Irr}(G)$ denote the set of all irreducible $\mathbf{C}G$-characters. In [DM1], Dong and Mason studied the sub-VOA $V^H=\{v\in V \mid h(v)=v \text{ for all } h\in H\}$ of H-invariants and the subspace V^χ on which G acts according to $\chi\in\mathrm{Irr}(G)$. Especially, they conjectured the following Galois correspondence between certain sub-VOAs of V and subgroups of G, which is the origin of their title of [DM1]. They proved it for an abelian or dihedral group G (see [DM1, Theorem 1]) and later for nilpotent groups (see [DM2]).

CONJECTURE 1 (Quantum Galois theory). *Let V be a simple VOA, and let G be a finite and faithful group of automorphisms of V. Then there is a bijection between the subgroups of G and the sub-VOAs of V which contain V^G defined by the map $H\to V^H$.*

Our purpose in this paper is to prove the above conjecture. Namely, we prove the following theorem.

THEOREM 1. *Let V be a simple VOA and let G be a finite and faithful group of automorphisms of V. Then there is a bijection between the subgroups of G and the sub-VOAs of V which contain V^G defined by the map $H\ (\leq G)\to V^H\ (\supseteq V^G)$.*

We adopt the notation and results in [DM1] and [DLM]. Especially, the following result in [DLM] is the main tool for our study.

THEOREM 2 [DLM, Corollary 2.5]. *Suppose that V is a simple VOA and that G is a finite and faithful group of automorphisms of V. Then the following hold.*

(i) *For $\chi\in\mathrm{Irr}(G)$, each V^χ is a simple module for the G-graded VOA $\mathbf{C}G\otimes V^G$ of the form*

$$V^\chi = M_\chi\otimes V_\chi,$$

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