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BLOW-UP RESULTS FOR NONLINEAR PARABOLIC EQUATIONS ON MANIFOLDS

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1. Introduction. The aim of this paper is threefold. First, by a unified approach, we prove that several classical blow-up results obtained over the last three decades for semilinear and quasilinear parabolic problems in $\mathbf{R}^n$ are valid on noncompact, complete Riemannian manifolds, which include those with nonnegative Ricci curvatures. Next, we remove some unnecessary a priori growth conditions on solutions of the quasilinear case, which are assumed in the existing literature. Finally, we demonstrate a new critical phenomenon for some inhomogeneous, quasilinear parabolic equations. We also hope that this paper serves as a link for the many other papers on this subject, which lie scattered in several journals over a period of three decades.

Specifically, we study the blow-up properties of the following homogeneous and inhomogeneous, semilinear parabolic equations and of the porous medium equations with nonlinear source:

$$(1.1)\qquad \begin{cases} \Delta u - \partial_t u + V(x)u^p = 0 & \text{in } \mathbf{M}^n \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbf{M}^n, u \geq 0, \end{cases}$$

$$(1.2)\qquad \begin{cases} \Delta u - \partial_t u + V(x)u^p + w = 0 & \text{in } \mathbf{M}^n \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbf{M}^n, u \geq 0, \end{cases}$$

$$(1.3)\qquad \begin{cases} \Delta u^{1+\sigma} - \partial_t u + u^p = 0 & \text{in } \mathbf{M}^n \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbf{M}^n, u \geq 0, \end{cases}$$

$$(1.4)\qquad \begin{cases} \Delta u^{1+\sigma} - \partial_t u + u^p + w = 0 & \text{in } \mathbf{M}^n \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbf{M}^n, u \geq 0, \end{cases}$$

where $\mathbf{M}^n$, with $n \geq 3$, is a noncompact complete Riemannian manifold, Δ is the Laplace-Beltrami operator, and $w = w(x) \geq 0$ is an L^1_{loc} function.

As we see later, there is a significant difference between the homogeneous problem ((1.1) and (1.3)) and the inhomogeneous ((1.2) and (1.4)) to warrant the separate listing. Homogeneous problems like (1.1) and (1.3) have been studied widely. In 1966, Fujita [F] proved the following results for (1.1) when $\mathbf{M}^n = \mathbf{R}^n$ and $V = 1$:

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