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RAMIFIED DEFORMATION PROBLEMS

BRIAN CONRAD

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0. Introduction. The proof of the semistable Taniyama-Shimura conjecture by Wiles [24] and Taylor-Wiles [23] uses as its central tool the deformation theory of Galois representations. In [6], Diamond extends these methods, proving that an elliptic curve $E_{/\mathbf{Q}}$ is modular if it is either semistable at 3 and 5 or is just semistable at 3, provided that the representation

$$\overline{\rho}_{E,3} : \mathrm{Gal}\left(\overline{\mathbf{Q}}/\mathbf{Q}\left(\sqrt{-3}\right)\right) \to \mathrm{Aut}\left(E[3](\overline{\mathbf{Q}})\right) \simeq \mathrm{GL}_2(\mathbf{F}_3)$$

is absolutely irreducible. His proof relies on extending the scope of the deformation-theoretic tools. The remaining obstacle to having an unconditional proof of the

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