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THE CORRESPONDENCES OF INFINITESIMAL CHARACTERS FOR REDUCTIVE DUAL PAIRS IN SIMPLE LIE GROUPS

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§1. Introduction. In the study of Howe correspondences between representations of real reductive dual pairs, one of the first important things that one wishes to know is the correspondence of infinitesimal characters. In the classical case of dual pairs in the symplectic group, such a correspondence has been known for a while (see [Ho4] and [P]). The purpose of this paper is to determine this correspondence for most dual pairs in simple Lie groups which are known to exist. For all the cases considered, we determine exactly which dual pairs admit the Howe duality phenomenon.

More precisely, we do the following. In classical groups, all reductive dual pairs are obtained by "direct sums," "tensor products," and combinations of the two constructions. We compute the correspondence for all such dual pairs in §4. What emerges is the fact that either there is no correspondence or the correspondence reduces to the familiar case of dual pairs in C_n (with the exception of dual pairs in $\mathfrak{so}(3, 2n-2)$ and $\mathfrak{so}(2, 2n-1)$). For dual pairs in A_n, the correspondence exists for both tensor product and direct sum types. For D_n, it exists only for direct sum type dual pairs, and a few very special tensor product type dual pairs. In either case, we are able to completely determine not only the correspondence of infinitesimal characters, but also the correspondence of representations.

For type B_n, the minimal representation exists only for the universal covering groups of $SO_0(3, 2n-2)$ and $SO_0(2, 2n-1)$. In these two cases, the minimal representation is definitely not obtainable from the oscillator representation of the symplectic group. Yet we can "pretend" it is and obtain the correct correspondence of infinitesimal characters of dual pairs (cf. the end of §4). We omit the discussion for dual pairs in C_n since the theory is well known.

The generalization of the direct sum construction is "removing a simple root from the extended Dynkin diagram." What remains after such removal (in most cases) are two connected components corresponding to two simple groups. These two groups usually form a dual pair. In §2 and §6, we consider all but one such dual pair in exceptional groups. (The excepted case is (A_1, A_3) in F_4). Somewhat surprisingly, a Howe correspondence exists here exactly when we are (more or less) in the "α" and "δ" cases studied by Rallis and Schiffmann [RS2]. That is, the root taken out is either α, a simple root connected to the highest root, or δ, the simple root connected

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