

DIAMETER RIGIDITY OF SPHERICAL POLYHEDRA

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1. Introduction. The diameter rigidity question considered in this paper is motivated by the rank rigidity problem for spaces of nonpositive curvature. The rank of a complete, simply connected space Y of nonpositive curvature is greater than or equal to 2 if every geodesic segment in Y is contained in an isometrically embedded, convex Euclidean plane. The rank rigidity problem asks for a classification of such spaces, at least when the isometry group of Y is large.

Recall that a topological space is called a polyhedron if it admits a triangulation. A polyhedron with a length metric is called Euclidean (respectively, spherical) if it admits a triangulation into Euclidean (respectively, spherical) simplices. Here a Euclidean (respectively, spherical) k -simplex is a k -simplex A such that A with the induced length metric is isometric to the intersection of $k + 1$ closed half-spaces in \mathbb{R}^k (respectively, closed hemispheres in S^k) in general position. We are interested in the rank rigidity of simply connected Euclidean polyhedra of nonpositive curvature and of rank greater than or equal to 2. We expect them to be Euclidean buildings or products. The rank rigidity for $\dim Y = 2$ is not difficult and is contained in [BB, Section 6].

For a Euclidean polyhedron Y and $p \in Y$, we denote by $S_p Y$ the link of Y at p , that is, the set of directions at p . Clearly, $S_p Y$ is a spherical polyhedron, and, if Y has nonpositive curvature, then the injectivity radius of $S_p Y$ is π . Furthermore, if the rank of Y is greater than or equal to 2, then $S_p Y$ is geodesically complete and has diameter π . Hence, for $\dim Y = 3$ the link $X = S_p Y$ is a geodesically complete, compact, 2-dimensional spherical polyhedron of diameter and injectivity radius π . The aim of this paper is to classify such spaces X . Here are examples of geodesically complete compact spherical polyhedra of diameter and injectivity radius π .

1.1. Spherical building. If X is a spherical building, then X carries a natural metric, for which the apartments are unit spheres. For this metric, the diameter and injectivity radius of X are π . If Y is a Euclidean building of dimension $n \geq 2$ with the natural metric, then every geodesic in Y is contained in an isometrically embedded, convex Euclidean n -space. The link of a vertex in Y is a spherical building of dimension $n - 1$, which has injectivity radius and diameter π . An n -dimensional building X

Received 14 June 1996. Revision received 2 December 1997.

1991 *Mathematics Subject Classification.* 53C20, 20F55, 51F15, 52B99, 57Q99.

Ballmann partially supported by Sonderforschungsbereich Theoretische Mathematik number 256 and the University of Maryland.

Brin partially supported by Sonderforschungsbereich Theoretische Mathematik number 256 and National Science Foundation grant number DMS-9504135.