RIGID COHOMOLOGY AND INVARIANT CYCLES FOR A SEMISTABLE LOG SCHEME

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Introduction. Let k be a perfect field of characteristic $p > 0$. We indicate by W the ring of Witt vectors of k and by K_0 its quotient field.

The study of cohomological theories, like rigid or crystalline cohomology, encounters some problems when applied to singular k-varieties. In this paper, we deal with the rigid cohomology of a particular type of singular k-varieties. Recently, a new theory, "log-crystalline" cohomology, has provided a way to associate to a proper, semistable (k, L)–log scheme (Y, M) the cohomology groups

$$H^i\big((Y, M)/(W, L)\big), \quad i \geq 0, \tag{0.1}$$

which are W-modules of finite type, endowed with nilpotent endomorphisms N_i (called the monodromy). (Here (k, L) means $\operatorname{Spec} k$ endowed with the pre–log structure $\mathbf{N} \to k$, $1 \to 0$; see [HK].)

In this paper we wish to show how the rigid cohomology of Y (which is singular, in general) is related to the kernel of the monodromy (i.e., the invariant cycles). In particular our conjecture is the following. Let (Y, M) be a proper, semistable (k, L)-log scheme. Then for each $i \geq 0$ there is an exact sequence

$$(*) \quad H^i_{\text{rig}}(Y/K_0) \to H^i\big((Y, M)/(W, L)\big) \otimes K_0 \xrightarrow{N_i \otimes K_0} H^i\big((Y, M)/(W, L)\big) \otimes K_0.$$

In the case $i = 1$, one actually has the exact sequence

$$0 \to H^1_{\text{rig}}(Y/K_0) \to H^1\big((Y, M)/(W, L)\big) \otimes K_0 \xrightarrow{N_1 \otimes K_0} H^1\big((Y, M)/(W, L)\big) \otimes K_0.$$

This is the p-adic analogue of the Lefschetz invariant cycles theorem in the complex case (see [I2] and [St]).

Consider a proper W-scheme X with semistable reduction and generic fiber X_{K_0}. Then the special fiber Y of X is endowed with a log structure M, which makes (Y, M) a proper, semistable (k, L)-log scheme. By [HK], our conjecture indicates that for each $i \geq 0$ there should be an exact sequence

$$H^i_{\text{rig}}(Y/K_0) \to H^i_{\text{dR}}\big(X_{K_0}/K_0\big) \xrightarrow{N_i \otimes K_0} H^i_{\text{dR}}\big(X_{K_0}/K_0\big),$$

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