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MAXIMAL OPERATORS OVER ARBITRARY SETS OF DIRECTIONS

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§0. Introduction. In this paper, with Ω a set of unit vectors in $\mathbb{R}^2$ of cardinality N, we study the maximal operator

$$\big(M_\Omega f\big)(x,y) = \sup_{r\in\mathbb{R}^+, v\in\Omega} \frac{1}{2r}\int_{-r}^{r} \big|f\big((x,y)+tv\big)\big|\,dt.$$

The best previously known L^2 bounds for this operator are due to Barrionuevo in [Ba], where it is shown that

$$||M_\Omega f||_{L^2} \le C N^{2/\sqrt{\log N}} ||f||_{L^2}.$$

The L^2 bounds for M_Ω imply L^4 bounds for the Fourier multiplier by the characteristic function of each of the polygons, which has the property that the normal direction of each side is contained in Ω. This is a result of Cordoba and Fefferman [CF].

Strömberg showed in [St] that if Ω is an equidistributed set of directions, then M_Ω satisfies the sharp estimate

$$||M_\Omega f||_{L^2} \le C \log N ||f||_{L^2}.$$

We establish that the same holds for every set Ω.

Let $\beta = (\beta_1, \beta_2)$ be any map from $\mathbb{R}^2$ to $\mathbb{R}^+ \times \Omega$. Then we may define the linearized maximal operator

$$\big(M_\beta f\big)(x,y) = \frac{1}{\beta_1(x,y)}\int_{-\beta_1(x,y)}^{\beta_1(x,y)} f\big((x,y)+t\beta_2(x,y)\big)\,dt.$$

In Section 1, we prove the following theorem.

THEOREM 1. *Let $E \subset \mathbb{R}^2$ be any set. Then there exists $C > 0$ a universal constant so that for any choice of β,*

$$\big\|M_\beta^* \chi_E\big\|_{L^2} \le C\sqrt{\log N}\,|E|^{1/2}.$$

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