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LEFSCHETZ CLASSES ON ABELIAN VARIETIES

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CONTENTS

Introduction. Let $\sim$ be an adequate equivalence relation on algebraic cycles, for example, rational equivalence (rat), homological equivalence with respect to some Weil cohomology theory (hom), or numerical equivalence (num). For a smooth projective variety X, $\mathscr{Z}^s(X)$ denotes the group of algebraic cycles on X of codimension s, and

$$\mathscr{C}^s_\sim(X) = (\mathscr{Z}^s(X)/\sim) \otimes \mathbb{Q}.$$

Then $\mathscr{C}_\sim(X) \overset{\text{df}}{=} \oplus_s \mathscr{C}^s_\sim(X)$ becomes a graded $\mathbb{Q}$-algebra under the intersection product, and we define $\mathscr{D}_\sim(X)$ to be the $\mathbb{Q}$-subalgebra of $\mathscr{C}_\sim(X)$, generated by the divisor classes

$$\mathscr{D}_\sim(X) = \mathbb{Q}[\mathscr{C}^1_\sim(X)].$$

The elements of $\mathscr{D}_\sim(X)$ are called the *Lefschetz classes* on X (for the relation $\sim$). They are the algebraic classes on X that can be expressed as linear combinations of intersections of divisor classes (including the empty intersection X).

Our main theorem states that, for any Weil cohomology theory $X \mapsto H^*(X)$ and any abelian variety A over an algebraically closed field, there is a reductive algebraic group $L(A)$ (not necessarily connected) such that the cycle class map induces an isomorphism

$$\mathscr{D}^s_{\text{hom}}(A^r) \otimes_{\mathbb{Q}} k \to H^{2s}(A^r)(s)^{L(A)}$$

for all integers $r, s \geqslant 0$; moreover, $\mathscr{D}^s_{\text{num}}(A^r) = \mathscr{D}^s_{\text{hom}}(A^r)$. Here $A^r = A \times \cdots \times A$ (r copies), k is the coefficient field for the cohomology theory, and s denotes a

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