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CRITICAL VALUES OF THE TWISTED TENSOR L-FUNCTION IN THE IMAGINARY QUADRATIC CASE

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1. Introduction. The twisted tensor L-function of f, which we denote by $G(s,f)$, is a certain Dirichlet series associated to a quadratic extension of number fields K/F and a cuspidal automorphic function f over K. It was introduced in [1] by Asai, following previous work of Shimura, in the case when f is a Hilbert modular cusp form over a real quadratic extension K of $\mathbf{Q}$.

In the past twenty-odd years, this L-function has been considered more generally: for instance, [11] and [12] deal with quadratic extensions of totally real fields, [17] with imaginary quadratic extensions of $\mathbf{Q}$, and [3], [4, appendix], and [14] with general quadratic extensions of number fields. All these papers have been primarily concerned with establishing analytic properties of $G(s,f)$ analogous to those in [1], such as meromorphic continuation to the entire complex plane, location and finiteness of the number of poles, and functional equation.

The aim of this paper is to prove a rationality result for $G(s,f)$ in the imaginary quadratic setting. If K is an imaginary quadratic field and f a cusp form associated to K, we establish that there is a *period* $\Omega_j(f)$ such that

$$\frac{G(j,f)}{\Omega_j(f)} \in E$$

for a finite set of integers j depending on the weight of f. Here E is a finite extension of the number field generated by the Fourier coefficient of f. Moreover, the period $\Omega_j(f)$ is the product of a fixed (probably transcendental) constant $\Omega(f)$, a power of $2\pi i$ depending on j, and a certain Gauss sum.

If we assume that $G(s,f)$ is motivic, we may interpret our result in the framework of Deligne's famous conjectures on motivic L-functions, outlined in [2]. Indeed, assuming that there is a rank-2 motive associated to f defined over K, the integers j above are the critical integers (in the sense of [2]) in the right half of the critical strip of the appropriately defined motive, whose L-function is $G(s,f)$. Moreover, the general shape of the period $\Omega_j(f)$ is predicted by this conjectural framework (see Proposition 3 and Remarks 3 and 4).

Unfortunately, since it is only conjectured that $G(s,f)$ is motivic, we cannot realize $\Omega_j(f)$ as a motivic period. Rather, we follow an alternative program, outlined by Hida in [10], to define a period independently of the motivic setting.

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