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ON THE UNITARY DUAL OF REAL REDUCTIVE LIE GROUPS AND THE $A_{\mathfrak{q}}(\lambda)$ MODULES: THE STRONGLY REGULAR CASE

SUSANA A. SALAMANCA-RIBA

1. Introduction. The classification of the irreducible unitary representations of a Lie group G is a very interesting problem, not only from the point of view of the physical applications, but also because of its importance in many areas of mathematics. (See, for example, [23]). However, even though this problem has been solved for nilpotent groups by Kirillov [9] and for type I solvable groups by Auslander and Kostant [1], the reductive case is not completely understood.

In [19] Vogan and the author propose a way of reducing this problem to the classification of a part of the unitary dual of a very special type of subgroup. That is, we decompose the set $\Pi_u(G)$ of nonequivalent, irreducible, unitary representations of G into disjoint sets parametrized by a discrete set of parameters $\{\lambda_u\}$. To a given representation, we associate a canonical subgroup determined by a given parameter λ_u in a way similar to the one used in [20] to parametrize the irreducible, admissible representations of G. The choice of this subgroup was inspired by the ideas in [20], [22], and [23], and by the author's previous work [16], [17], and [18].

In this paper we use this approach to prove an old conjecture of Vogan and Zuckerman (stated in Theorem 1.8) on the classification of certain unitary representations of a real reductive Lie group G. The search for a proof of this conjecture is what motivated the program described in [19].

We first need some terminology and some facts before we state the theorem. In order to expedite the statement of the result and the presentation of the arguments involved in the proof, we go over this background material quickly and without explanation in this introduction. This is general background information on representation theory of Lie groups, especially on unitary representations. Some of this background is briefly explained in Section 2 for completeness. The reader who is familiar with it may skip that preamble; however, we assume knowledge of structure theory, representation theory of compact Lie groups, and finite-dimensional representation theory of Lie groups and algebras.

To simplify matters in this section and the next, we assume that G is a real, connected, semisimple Lie group with finite center. We use the same notation as in [19]. Some of it is recorded here for convenience. Let $\mathfrak{g}_0$ be the Lie algebra of

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