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A FAMILY OF STRATIFIED AREA-MINIMIZING CONES

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1. Introduction. In this paper, we use a geometric method called directed slicing, introduced in [L2], to show that certain minimal cones are area minimizing. To apply this method to a minimal surface M, we slice the space surrounding M into leaves L of a (possibly singular) "foliation." In doing so, we obtain a family of minimization problems, one on each leaf. If we can show that each slice $L \cap M$ of M minimizes a particular measure within its respective leaf L, then we may conclude that M itself is (globally) area minimizing. The measure on each leaf is determined by a "weighting function" w, which is basically the cross-sectional area of a small neighborhood of slicing sets and is computed via Jacobians.

An outline of the remainder of this paper follows. In Section 2, we fix terminology and notation for directed slicing. In Section 3, we introduce the idea of normal space slicing and establish a general formula for weighting functions associated to slicings constructed in this way. Next we discuss geometric properties of the cone of p by q matrices of rank at most r. Section 5 contains an overview of Sections 6 and 7, wherein the details of our construction of slicing sets for a neighborhood of this cone and the computation of the associated weighting function may be found. We then verify that each slice of the cone solves the corresponding minimization problem within its respective slicing set. This verification is done by slicing again. The main result of the paper—that cones with $p + q - 2r \geqslant 4$ are area minimizing—is stated and proved in Section 8.

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2. Notation and terminology for directed slicing

Definition 2.1. Let M be a k-dimensional surface (rectifiable current or rectifiable current reduced modulo 2) in $\mathbf{R}^n$. For $0 \leqslant d < k$, a *d-dimensional slicing* of M is a collection of pairwise disjoint, d-dimensional rectifiable subsets of M, and a *full d-dimensional slicing* of M is a d-dimensional slicing of M with the property that the union of its subsets covers $\mathscr{H}^k$-almost all of M.

Typically, we are concerned with an $(n - k + d)$-dimensional slicing of a neighborhood of M in $\mathbf{R}^n$ with the property that the collection of intersections of

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