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FINITE ENERGY SURFACES AND THE CHORD PROBLEM

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CONTENTS

1. Introduction. A *contact form* on an odd-dimensional manifold M of dimension $2n+1$ is a 1-form λ such that the $(2n+1)$-form Ω, given by

$$\Omega = \lambda \wedge (d\lambda)^n,$$

defines a volume form on M. We observe that any manifold admitting a contact form is necessarily orientable and that a contact form defines a natural orientation.

Assume now that (M, λ) is a manifold together with a given contact form. First of all, we note that λ defines a $2n$-dimensional vector bundle over M. Indeed, consider $\xi \to M$, where ξ is given by

$$\xi_m = \ker(\lambda_m).$$

The linear functional $\lambda_m : T_mM \to \mathbf{R}$ is nonzero since $\lambda \wedge (d\lambda)^n$ defines a volume form. Hence we obtain a vector bundle. Moreover, by the properties of λ, we see that $\omega := d\lambda \,|\, (\xi \oplus \xi)$ is nondegenerate on each fibre. Clearly, $\omega : \xi_m \oplus \xi_m \to \mathbf{R}$ is also skew-symmetric and bilinear; hence it is a symplectic form on ξ_m. Therefore, (ξ, ω) is a symplectic vector bundle.

Since the dimension of M is odd, $d\lambda$ is degenerate on each fibre T_mM of the tangent bundle TM. But it is as good as it can be, since λ is a contact form. Therefore, we obtain a line bundle ℓ over M via the definition

$$\ell_m = \{p \in T_mM \,|\, d\lambda_m(p, q) = 0 \text{ for all } q \in \xi_m\}.$$

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