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KOSTANT POLYNOMIALS AND THE COHOMOLOGY RING FOR G/B

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The Schubert calculus for G/B can be completely determined by a certain matrix related to the Kostant polynomials introduced in [1, Sect. 5]. The polynomials are defined by vanishing properties on the orbit of a regular point under the action of the Weyl group. For each element w in the Weyl group, the polynomials also have nonzero values on the orbit points corresponding to elements that are larger than w in the Bruhat order. Our main theorem is an explicit formula for these values. The matrix of orbit values can be used to determine the cup product for the cohomology ring for G/B, using only linear algebra or as described in [14].

1. Introduction. Let G be a semisimple Lie group, H be a Cartan subgroup, W be its corresponding Weyl group with generators $\sigma_1, \sigma_2, \ldots, \sigma_n$, and B be a Borel subgroup. Let $\mathbb{C}[\mathbf{h}^*]$ be the algebra of polynomial functions on the Cartan subalgebra $\mathbf{h}$ over $\mathbb{C}$. Fix a regular element $\mathbf{O} \in \mathbf{h}$ such that $\alpha_i(\mathbf{O})$ is a positive integer for all simple roots α_i. Any Weyl group element v acts on the right on $\mathbf{O}$ by the action on the Cartan subalgebra. We define the following interpolating polynomials by their values on the orbit of $\mathbf{O}$.

Definition 1. A *Kostant polynomial* K_w is any element of $\mathbb{C}[\mathbf{h}^*]$ of degree $l(w)$ (nonhomogeneous) such that

$$K_w(\mathbf{O}v) = \begin{cases} 1, & v = w, \\ 0, & l(v) \leqslant l(w) \text{ and } v \neq w. \end{cases} \tag{1.1}$$

These polynomials were defined originally by Kostant and appear in [1, Thm. 5.9] for the finite case. They were later generalized by Kostant and Kumar in [14], there denoted $\xi_{w^{-1}}$. Kostant showed that K_w is unique modulo the ideal of all elements of $\mathbb{C}[\mathbf{h}^*]$ that vanish on the orbit of $\mathbf{O}$ under the Weyl group action. Furthermore, he showed that the highest homogeneous component of a Kostant polynomial represents a Schubert class in the cohomology ring of G/B. Indeed, Carrell has shown there is a direct connection between the ring of polynomials defined on the orbit $\mathbf{O}W$ and the cohomology ring of G/B. Namely, $H^*(G/B)$ is isomorphic to the graded ring canonically associated to the polynomial ring of

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