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RESOLVING MIXED HODGE MODULES ON CONFIGURATION SPACES

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If $\pi : X \to S$ is a continuous map of locally compact topological spaces and n is a natural number, let X^n/S be the nth fibred power of X with itself. Let $\mathsf{F}(X/S, n)$ be the configuration space whose fibre $\mathsf{F}(X/S, n)_s$ over a point $s \in S$ is a configuration of n distinct points in the fibre X_s, or equivalently, the complement of the $\binom{n}{2}$ diagonals in X^n/S. Let $j(n) : \mathsf{F}(X/S, n) \hookrightarrow X^n/S$ be the natural open embedding.

An essential role is played in this paper by the higher direct image with compact support $f_!$ (this is often written $\mathsf{R}f_!$) associated to a continuous map $f : X \to Y$. This is a functor from the derived category of sheaves of abelian groups on X to the derived category of sheaves of abelian groups on Y. For example, if Y is a point and $\mathscr{F}$ is a sheaf on X, then $f_!\mathscr{F}$ is a complex of abelian groups whose cohomology is $H_c^\bullet(X, \mathscr{F})$. If f is either a closed or an open embedding, $f_!$ takes sheaves on X to sheaves on Y. (See Section 1 of Verdier [22].)

Given a sheaf $\mathscr{F}$ of abelian groups on X^n/S, we introduce a natural resolution $\mathscr{L}^\bullet(X/S, \mathscr{F}, n)$ of the sheaf $j(n)_! j(n)^* \mathscr{F}$ by sums of terms of the form $i(J)_! i(J)^* \mathscr{F}$. (Here, $i(J)$ is the closed embedding of a diagonal in X^n/S.) This resolution has the property that if $\mathscr{F}$ is an $\mathbb{S}_n$-equivariant sheaf (where the symmetric group $\mathbb{S}_n$ acts on X^n/S by permuting the factors in the fibred product), the resolution is $\mathbb{S}_n$-equivariant as well. For example, if $n = 2$, we have the exact sequence of sheaves

$$0 \to j(2)_! j(2)^* \mathscr{F} \to \mathscr{F} \to i_! i^* \mathscr{F} \to 0, \tag{0.1}$$

where $i : X \hookrightarrow X^2/S$ is the diagonal embedding. When X is a Riemann surface, this resolution was introduced by Looijenga [13].

Let $\pi(n) : \mathsf{F}(X/S, n) \to S$ and $\pi(n) : X^n/S \to S$ be the projections to S. (We denote them by the same symbol, since confusion is hardly likely to arise.) The objects $\pi(n)_! j(n)^* \mathscr{F}$ and $\pi(n)_! \mathscr{L}^\bullet(X/S, \mathscr{F}, n)$ are isomorphic in the derived category of sheaves on S. We use this isomorphism to calculate the $\mathbb{S}_n$-equivariant Euler characteristic of $\pi(n)_! j(n)^* \mathscr{F}$.

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