

SEMICLASSICAL EIGENVALUE ESTIMATES FOR
THE PAULI OPERATOR WITH STRONG
NONHOMOGENEOUS MAGNETIC FIELDS,
I: NONASYMPTOTIC LIEB-THIRRING-TYPE ESTIMATE

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1. Introduction. In this paper and its companion [ES], we study the semi-classical limit of the Pauli operator with both electric and magnetic fields. Our main concern is to allow for nonhomogeneous magnetic fields. As we hope to illustrate, the transition from homogeneous to nonhomogeneous fields is highly nontrivial. This is so not only because of technical difficulties, but also because the case of nonhomogeneous fields is qualitatively different.

Received 30 May 1996.

1991 *Mathematics Subject Classification*. Primary 35P15, 35J10; Secondary 81Q20.