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COHOMOLOGICAL INDUCTION IN VARIOUS CATEGORIES AND THE MAXIMAL GLOBALIZATION CONJECTURE

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1. Introduction. Cohomological induction is a major means that representation theorists use to construct representations. It is a purely algebraic construction. There is an obvious analytic counterpart, using Dolbeault cohomologies. For a long time, it has been conjectured that (e.g., [V1]) the Dolbeault cohomologies and the cohomologically induced modules are "essentially" the same. We prove this conjecture in this paper.

This conjecture has been referred to without having a name. Perhaps part of the reason is that it has had a long history of evolution and different people have made different inputs. It is after a long, cumulative process that the conjecture appears in its final and precise form, as stated in Theorem 2.4. For the sake of convenience, we call (thanks to a suggestion from D. Vogan) this conjecture the "maximal globalization conjecture."

Since the maximal globalization conjecture has a rich history, it is appropriate to recount it briefly. This is not meant to be an exhaustive and complete account; interested readers can consult either [KV] or [Wo1], among others, for more details. The first significant example of cohomological induction is the Borel-Weil-Bott construction, in which case the coincidence between the algebraic and analytic construction is basically trivial, as the domain is compact. Next, Schmid constructed the discrete series representations analytically in [S1]. The major hurdle to surmount in the construction is to show that, with respect to the usual Fréchet topology, the differentials of the Dolbeault complex have closed ranges or else we do not even have good topologies for the representations. Zuckerman then considered the algebraic analogue for Schmid's construction, which can be carried out in a much wider setting (see [V1] and [V2], for example). We call the algebraic construction the *Vogan-Zuckerman modules.* The papers [AR], [S2], and [SW] contain results about which topological completion of the underlying Harish-Chandra module is provided by the Dolbeault cohomologies, in case the Dolbeault complex has the closed range property. In [Wo2], the coincidence between the Dolbeault cohomologies and Vogan-Zuckerman modules is established for the finite rank vector bundle case. The method makes use of a general procedure called the Penrose transform, which [S1] also uses. In [W2], the case of certain infinite rank bundles is settled.

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