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FLOWS ON HOMOGENEOUS SPACES AND DIOPHANTINE PROPERTIES OF MATRICES

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Introduction

Notation. We denote by $M_{m,n}(\mathbb{R})$ the space of real matrices with m rows and n columns. $I_k \in M_{k,k}(\mathbb{R})$ stands for the identity matrix. Vectors are named by lowercase boldface letters, such as $\mathbf{x} = (x_i \mid 1 \leqslant i \leqslant k)$, and, despite the row notation, are always treated as column vectors. Zero means a zero vector in any dimension, as well as a zero matrix of any size. For a matrix $L \in M_{m,n}(\mathbb{R})$ and $1 \leqslant i \leqslant m$, we denote by L_i the linear form $\mathbb{R}^n \to \mathbb{R}$ corresponding to the ith row of L, and by $L^{(i)}$ (resp., $L_{(i)}$) the matrix consisting of first (resp., last) i rows of L.

Any statement involving "$\pm$" stands for two statements, one for each choice of the sign. The Hausdorff dimension of a subset Y of a metric space X is denoted by $\dim(Y)$, and we say that Y is *thick* (in X) if, for any nonempty open subset W of X, $\dim(W \cap Y) = \dim(W)$ (i.e., Y has full Hausdorff dimension at any point of X).

In what follows, we fix two positive integers m and n, denote by G the group $\{L \in GL_{m+n}(\mathbb{R}) \mid \det(L) = \pm 1\}$ and by $\Omega \cong SL_{m+n}(\mathbb{R})/SL_{m+n}(\mathbb{Z}) \cong G/GL_{m+n}(\mathbb{Z})$ the space of unimodular lattices in $\mathbb{R}^{m+n}$.

History. A system $(A_1, \dots, A_m)$ of linear forms in n variables is called *badly approximable* if there exists a constant $c > 0$ such that for every $\mathbf{p} \in \mathbb{Z}^m$ and $\mathbf{q} \in \mathbb{Z}^n \setminus \{0\}$

$$\max(|A_1(\mathbf{q}) + p_1|^m, \dots, |A_m(\mathbf{q}) + p_m|^m) \cdot \max(|q_1|^n, \dots, |q_n|^n) > c\,. \tag{1}$$

W. Schmidt proved in 1969 [S3] that matrices $A \in M_{m,n}(\mathbb{R})$, such that the system $(A_1, \dots, A_m)$ is badly approximable, form a thick subset of $M_{m,n}(\mathbb{R})$.

In 1986, S. G. Dani exhibited a correspondence between badly approximable systems of linear forms and certain bounded trajectories in Ω. His result [D1, Theorem 2.20] can be restated as follows: For $A \in M_{m,n}(\mathbb{R})$, consider the lattice $\Lambda = \left(\begin{smallmatrix} I_m & A \\ 0 & I_n \end{smallmatrix}\right)\mathbb{Z}^{m+n} \in \Omega$ and the 1-parameter subgroup of G of the form

$$g_t = \operatorname{diag}(e^{t/m}, \dots, e^{t/m}, e^{-t/n}, \dots, e^{-t/n})\,. \tag{2}$$

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