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NONSYMMETRIC JACK POLYNOMIALS AND INTEGRAL KERNELS

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1. Introduction. Nonsymmetric Jack polynomials occur as the polynomial part of the eigenfunctions of the Calogero-Sutherland model on a circle with exchange terms. This means, in particular, that they are eigenfunctions of the transformed Hamiltonian

$$H^{(C)} = \sum_{j=1}^{n} x_j^2 \frac{\partial^2}{\partial x_j^2} + \frac{2}{\alpha} \sum_{1 \leqslant j<k \leqslant n} \frac{x_j x_k}{x_j - x_k} \left[\left(\frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_k} \right) - \frac{1 - s_{jk}}{x_j - x_k} \right]. \tag{1.1}$$

Here s_{jk} is the operator that acts on functions by exchanging the jth and kth coordinates. The nonsymmetric Jack polynomials $E_\eta(x)$, $x := (x_1, \ldots, x_n)$ were introduced by Opdam [28] and their properties expounded upon in [18], [29]. (Their q-analogues, the nonsymmetric Macdonald polynomials, were introduced in [23] and have also received attention in the literature; see [4], [25].) In what follows, we mainly follow the notation of Knop [18] and Sahi [29].

The $E_\eta(x)$ are labelled by an n-tuple $\eta = (\eta_1, \eta_2, \ldots, \eta_n) \in \mathbb{N}^n$ and are uniquely defined as being the simultaneous eigenfunctions of the (mutually commuting) Cherednik operators ξ_i defined by

$$\xi_i = \alpha x_i \frac{\partial}{\partial x_i} + \sum_{p<i} \frac{x_i}{x_i - x_p}(1 - s_{ip}) + \sum_{p>i} \frac{x_p}{x_i - x_p}(1 - s_{ip}) + 1 - i \tag{1.2}$$

and by the fact that they have an expansion of the form

$$E_\eta(x) = x^\eta + \sum_{\nu \prec \eta} a_{\eta\nu} x^\nu . \tag{1.3}$$

Here the partial order $\prec$ on n-tuples is defined for $\eta \neq \nu$ by $\nu \prec \eta$ if and only if $\nu^+ < \eta^+$, or in the case $\nu^+ = \eta^+$, $\nu < \eta$, where η^+ is the unique partition associated with η obtained from permuting its entries and $<$ is the usual dominance order for n-tuples; that is, $\nu < \eta$ if and only if $\sum_{i=1}^{p}(\eta_i - \nu_i) \geqslant 0$, for all $1 \leqslant p \leqslant n$.

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