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# EMBEDDINGS OF $PGL_2(31)$ AND $SL_2(32)$ IN $E_8(\mathbb{C})$

ROBERT L. GRIESS, JR. AND A. J. E. RYBA
APPENDIX A BY MICHAEL LARSEN
APPENDIX B BY J.-P. SERRE

CONTENTS



**0. Introduction.** In this paper we continue the program, started in [CG], to classify the finite simple subgroups of $E_8(\mathbb{C})$. In particular we show that $E_8(\mathbb{C})$ has three conjugacy classes of $PGL(2,31)$-subgroups and a single conjugacy class of $SL(2,32)$-subgroups. We obtain three nonconjugate embeddings of the simple group $PSL(2,31)$ in $E_8(\mathbb{C})$ from our embeddings of $PGL(2,31)$. Earlier work of [CGL] constructs $PSL(2,61)$ as a subgroup of $E_8(\mathbb{C})$, and work of [S3] constructs $PSL(2,61)$ and two of our three classes of $PGL(2,31)$-subgroups in $E_8(\mathbb{C})$. Our method is computational and should apply to the construction of other simple subgroups of $E_8(\mathbb{C})$. In future work, we plan to complete the determination of which finite simple groups are subgroups of $E_8(\mathbb{C})$ by classifying embeddings of $PSL(2,31)$, $PSL(2,41)$, $PSL(2,49)$, and $Sz(8)$ into $E_8(\mathbb{C})$.

In Section 1, we obtain an important collection of lifting and conjugacy theorems for finite subgroups of quasisimple algebraic groups. In particular, we show that if $L$ is a finite group and $p$ is a prime that does not divide $|L|$, then the number of conjugacy classes of $L$-subgroups in a quasisimple algebraic group in characteristic 0 is at most the number of conjugacy classes of $L$-subgroups in a corresponding algebraic group in characteristic $p$; we give sufficient conditions for equality. It turns out that an argument with algebraic geometry techniques proves that equality holds in general. This is due to Michael Larsen, who has contributed it as Appendix A. We have also included a letter from J.-P. Serre as Appendix B. In this

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