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SOLUTIONS OF SUPERLINEAR ELLIPTIC EQUATIONS AND THEIR MORSE INDICES, II

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1. Introduction. We are concerned with solutions of

$$\begin{cases} -\Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{I}$$

where Ω is a bounded, open, smooth domain in $\mathbb{R}^N (N \geqslant 2)$; $f(x,t)$ is continuous on $\bar{\Omega} \times \mathbb{R}$, differentiable with respect to t; and $\partial f/\partial t$ is continuous on $\bar{\Omega} \times \mathbb{R}$. We study here the superlinear case, that is,

$$\lim_{t \to \pm\infty} \frac{f(x,t)}{t} = +\infty, \qquad \text{uniformly in } x, \tag{H$_1$}$$

with a "subcritical" growth, that is,

$$|f(x,t)| \leqslant c(1 + |t|^p), \qquad c > 0. \tag{H$_2$}$$

Here p satisfies $1 < p < (N+2)/(N-2)$ if $N \geqslant 3$ and p is arbitrary in $(1,\infty)$ if $N = 2$. If $f(x,.)$ is odd for all $x \in \Omega$, then the existence of infinitely many solutions of (I) is known—see A. Ambrosetti and P. H. Rabinowitz [2]—and is obtained by variational arguments (so-called min-max principles).

The general case is much harder, and multiplicity results are known with some restrictions (on the growth and the form of f). See the work of A. Bahri and H. Berestycki [4], Bahri [3], Rabinowitz [13], and Bahri and P.-L. Lions [5].

In [5] multiplicity results are obtained by combining the "perturbative" approach introduced by Bahri and Berestycki in [4] with sharp estimates on "min-max values" found through an estimate for the "Morse index" of the associated critical points. Therefore, it seems natural to relate the Morse index of a solution of (I) with other qualitative properties. There are other deep reasons that motivate the search for links between the Morse index and the properties of the solutions: If we come back to the proof of the result of existence of infinitely many solutions by Rabinowitz in dimension 1 [12], we can see that he classifies the solutions, in the bifurcation diagram he studies, by the number of their zeros. These bifurcation branches are then bounded and cannot go from one eigenfunction to another, since they have distinct numbers of zeros. The result follows.

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