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THE CAUCHY RIEMANN EQUATION ON SINGULAR SPACES

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1. Introduction. The Cauchy Riemann operator $\bar{\partial}$ is a very useful tool in higher-dimensional complex analysis. Fundamental questions like the Levi problem have been solved using $\bar{\partial}$ on nonsingular complex manifolds. (See [FK], [HL], [H], [K], and [R] for background and references on the subject.) Complex spaces with singularities are the natural settings for many questions in complex analysis. But the $\bar{\partial}$-theory has been, so far, essentially restricted to nonsingular complex manifolds. One reason for this limitation is that it is difficult to define forms in the presence of singularities. The goal of this paper is to study the $\bar{\partial}$ equation on singular spaces addressing the fundamental problems of: defining the form on the variety; proving the existence of solutions to the $\bar{\partial}$ equation on the variety; and studying the smoothness of the solution in a neighborhood of the singularities.

There are basically three different approaches to these problems. One method is to blow up the singularity and to reduce the $\bar{\partial}$ problem to the resulting complex manifold. We are not considering this setup since our main goal is to solve the $\bar{\partial}$ equation on the original singular space. A second method, the extrinsic way, is to consider $\bar{\partial}$-closed forms α on some (variable) neighborhood of an embedded variety and estimate solutions to $\bar{\partial}u = \alpha$ on smaller neighborhoods of the variety. Some partial results are known in this setup. For example, let B be the closed unit ball in $\mathbb{C}^n$. If $g_1, \ldots, g_p$ are holomorphic functions in a neighborhood of B, then $M = \{z \in B : g_1(z) = \cdots = g_p(z) = 0\}$ is an analytic set in B. Henkin and Polyakov in [HP] (see also [HP] for references to the above approaches) proved the following theorem.

THEOREM 1.1. *Suppose that M is a complete intersection and that α is a differential form on B with coefficients of class $\mathscr{C}^\infty$ such that the coefficients of $\bar{\partial}\alpha$ vanish at all points of M. Then there exists a differential form β on $B \backslash Sing(M)$ with coefficients of class $\mathscr{C}^\infty$ such that $\bar{\partial}\beta = \alpha$ at each regular point of M.*

A general theory with this method is not possible. We give in Section 5 a counterexample that shows that a Hölder solution cannot be obtained from bounded data with the extrinsic definition.

The third approach, the intrinsic procedure, is to consider $\bar{\partial}$-closed forms α on the regular points of the variety and estimate solutions to $\bar{\partial}u = \alpha$ on the regular

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