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GEOMETRIC INTERPRETATION OF THE POISSON STRUCTURE IN AFFINE TODA FIELD THEORIES

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Introduction. Since the work of Zakharov and Shabat [23], dressing techniques have played an important role in the theory of classical integrable systems. These techniques were developed by Drinfeld and Sokolov in [7] in the framework of affine Toda field theories. Later, Feigin and Frenkel proposed in [13] and [14] another approach to these theories, which was shown in [8] and [10] to be equivalent to that of [7]. In those works, the space of local fields of the Toda theory (equivalently, the mKdV hierarchy) associated to an affine Lie algebra $\mathfrak{g}$ is described as the ring of functions on the coset space N_+/A_+ of a unipotent subgroup of the Kac-Moody group G corresponding to $\mathfrak{g}$. The mKdV flows are then identified with the right action of the principal commutative Lie algebra $\mathfrak{a}$ normalizing A_+. This identification leads to a system of variables in which the flows become linear, and hence can be integrated.

In the works on quantization of the Toda theories, an important role is played by the vertex operator algebra structure on the space of local fields. At the classical level, this gives rise to what we call here a vertex Poisson algebra (VPA) structure on the space of local fields of a Toda theory. The notion of the VPA structure coincides with the notion of "coisson algebra" (on the disc) introduced by Beilinson and Drinfeld in [5]. The goal of this work is to define this and related structures on the space of fields of a Toda theory in the Lie group terms using the identification described above.

To this end, we make use of an idea introduced earlier by Feigin and Enriquez in [9], where a similar problem was solved in the setting of the classical lattice Toda theory associated to $\widehat{\mathfrak{sl}}_2$. In that work, the space of local fields was extended by the "half screening charges" and "half integrals of motions." The screening charges and integrals of motions (IMs) are sums of the lattice translates of certain expressions, and their "halfs" are just the sums of positive translates of the same expressions. The full space was then identified with the quotient G/H, where H is the Cartan subgroup of G. The smaller spaces of fields, without half screening charges (resp., without half IMs), can be obtained by taking the quotient of G/H from the left by a Borel subgroup B_- (resp., from the right by the positive part of the loop group of the Cartan subgroup). In this interpreta-

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