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THE IDENTITY COMPONENT OF THE ISOMETRY GROUP OF A COMPACT LORENTZ MANIFOLD

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1. Introduction. The goal of this paper is to prove the following theorem.

THEOREM 1.1. *Let the affine group AG act isometrically on a compact Lorentz manifold* $(M, \langle\ ,\ \rangle)$. *Then some finite cover* $PSL_k(2,\mathbf{R})$ *of* $PSL(2,\mathbf{R})$ *acts isometrically on M. In fact, the initial action of AG is contained in an isometric action of* $PSL_k(2,\mathbf{R}) \times \mathbf{T}$, *where* $\mathbf{T}$ *is a torus of some dimension.*

This result may be compared with a theorem of E. Ghys [G1] (see also [Bel]) asserting a similar conclusion but assuming that M has dimension 3 and that the action is just volume preserving and locally free. The statement in [G1] is that the action of AG may be extended to an action of a finite cover of $PSL(2,\mathbf{R})$ or to an action of the solvable 3-dimensional Lie group SOL.

Here we have another motivation: to understand the structure of Lie groups acting isometrically on compact Lorentz manifold. The first results in the subject are due to [Zi], [Gr], and [D'A]. A "final" result is due to [AS] and [Ze1], independently. Necessary and sufficient conditions were given to see that a Lie group acts isometrically (and locally faithfully) on a compact Lorentz manifold. Note, however, that if a group acts in such a fashion, then its subgroups also act in the same way. For instance, all known examples of isometric actions of AG are obtained by viewing it as a subgroup of $SL(2,\mathbf{R})$. So a natural question is: What are the maximal (connected) Lie groups acting isometrically on a compact Lorentz manifold? Equivalently, we have the following question.

Question 1.2. What is the identity component $\mathrm{Isom}^0(M)$ of the isometry group of a compact Lorentz manifold M?

In dimension 3, a result of [Ze2] describes the geometric structure of a compact Lorentz manifold (of dimension 3) M, with $\mathrm{Isom}^0(M)$ noncompact. It has the following corollary.

THEOREM 1.3 [Ze2]. *If a compact Lorentz 3-manifold M has* $\mathrm{Isom}^0(M)$ *noncompact, then* $\mathrm{Isom}^0(M)$ *is isomorphic to* $\mathbf{R}$ *or to a finite cover of* $PSL(2,\mathbf{R})$.

Let us now recall the result of [AS] and [Ze1]. To simplify, we state the following results only on the Lie algebra level.

THEOREM 1.4 [AS] and [Ze1]. *Let G be a (connected) Lie group acting isometrically on a compact Lorentz manifold M. Then the Lie algebra* $\mathscr{G}$ *has a direct*

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