

LIFTING OF CHARACTERS ON ORTHOGONAL AND METAPLECTIC GROUPS

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§1. Introduction. An important principle in representation theory and automorphic forms is that of lifting or transfer of representations between reductive algebraic groups. Endoscopic transfer and base change are primary examples. Another type of example is provided by theta-lifting between members of a reductive dual pair. In this paper, we study lifting, defined directly on characters, between special orthogonal groups $SO(2n+1)$ over \mathbb{R} and the nonlinear metaplectic group $\widetilde{Sp}(2n, \mathbb{R})$. This is closely related both to endoscopy and theta-lifting, and is an aspect of the duality between root systems of types B_n and C_n .

Let π be an irreducible representation of $SO(p, q)$, the special orthogonal group of a symmetric bilinear form in $p+q=2n+1$ real variables; π has a non-zero theta-lift to a representation π' of $\widetilde{Sp}(2n, \mathbb{R})$. A natural question is, What is the relationship, if any, between the global characters of π and π' ? When $n=1$, this is closely related to the Shimura correspondence, which has been the subject of extensive study.

Evidence for such a relation is provided by the orbit correspondence, which induces a matching of semisimple conjugacy classes of $SO(p, q)$ and $Sp(2n, \mathbb{R})$. This is analogous to the matching of stable conjugacy classes in the theory of

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