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MULTIVARIABLE q-RACAH POLYNOMIALS

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The Koornwinder-Macdonald multivariable generalization of the Askey-Wilson polynomials is studied for parameters satisfying a truncation condition such that the orthogonality measure becomes discrete with support on a finite grid. For this parameter regime, the polynomials may be seen as a multivariable counterpart of the (one-variable) q-Racah polynomials. We present the discrete orthogonality measure and expressions for the normalization constants converting the polynomials into an orthonormal system (in terms of the normalization constant for the unit polynomial). We also discuss the limit $q \to 1$ leading to multivariable Racah type polynomials. Of special interest is the situation in which q lies on the unit circle. In that case, it is found that there exists a natural parameter domain for which the discrete orthogonality measure (which is complex, in general) becomes real-valued and positive. We investigate the properties of a finite-dimensional discrete integral transform for functions over the grid, whose kernel is determined by the multivariable q-Racah polynomials with parameters in this positivity domain.

1. Introduction. Some years ago, Koornwinder [K] extended a construction of Macdonald [M1] (see also [VK1]) to arrive at a multivariable generalization of a family of basic hypergeometric polynomials commonly known as the Askey-Wilson polynomials (see [AW2] and [GR]). The multivariable polynomials of interest depend rationally on a number of parameters and for parameter values in a certain domain. These polynomials form an orthogonal system with respect to an explicitly given (positive) continuous weight function with support on a (real) n-dimensional torus (where n denotes the number of variables). Recently, it was shown that the parameter domain, for which the multivariable Askey-Wilson polynomials admit such an interpretation as orthogonal polynomials, may be extended if one allows the corresponding orthogonality measure to have a partly continuous and partly discrete support (see [S]) (thus further generalizing the corresponding situation in the case of one single variable, where the phenomenon of discrete masses emerging in the Askey-Wilson orthogonality measure was already known to occur; see [AW2].)

In the present paper, we will demonstrate that for a different parameter regime satisfying a certain truncation condition, the multivariable Askey-Wilson polynomials can be reduced to a finite-dimensional orthogonal system with respect

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