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PARABOLIC SINGULAR INTEGRALS OF CALDERÓN-TYPE, ROUGH OPERATORS, AND CALORIC LAYER POTENTIALS

STEVE HOFMANN

Introduction. The theory of those multilinear and nonlinear singular integral operators (e.g., the Calderón commutators, the Cauchy integral on a Lipschitz curve, the double-layer potential for Laplace's equation on a Lipschitz graph) that arise in elliptic partial differential equations is now quite well understood; see, for example, papers by Calderón [Ca1] and [Ca2], David and Journé [DJ], Coifman, MacIntosh, and Meyer [CMM], Coifman and Meyer [CM], Christ and Journé [CJ, Sections 1–4], and Christ [Ch]. The fundamental modern tool for handling such operators is the "T1" theorem in [DJ], which permits one to control a k-linear operator by means of a $(k-1)$-linear operator, thus setting up a natural induction scheme. Furthermore, the T1 theorem permits one to view at least some nonlinear singular integral operators as a perturbation of the linear case; see, for example, [Ch].

In contrast, the multilinear and nonlinear singular integrals that arise in parabolic partial differential equations (PDEs) have not been nearly so well understood. These "parabolic" singular integrals are invariant with respect to a nonisotropic group of dilations, but it is not this "mixed homogeneity," per se, which causes problems: a well-developed theory of convolution operators of parabolic type has existed for some time; see, for example, papers by B. F. Jones [JoB], Fabes and Riviere [FR1] and [FR2], and Riviere [R]. Furthermore, a T1 theorem does exist in this setting, and even in the more general context of spaces of homogeneous type. (The result of R. Coifman has never been published; a more general result using, in part, the ideas of Coifman appears in a paper by David, Journé, and Semmes [DJS]. See also papers by Lemarie [L] and Lewis and Murray [LM1].) The real difficulty turns out to be that in the parabolic setting, one is forced to treat a class of singular integral operators (namely, those that arise when one applies a half-order time derivative to the "smoothing operators of Calderón-type," such as the caloric single-layer potential) that are "rough" in some sense. In particular, the fundamental premise of the T1 theorem, namely, that $T1 \in BMO$, may fail. Thus, for these operators, a direct attack via the nonisotropic T1 theorem does not work. However, we shall see that a local substitute for the condition that $T1 \in BMO$ does hold. This condition is a sort of "quasi-Carleson measure condition" and is similar to a condi-

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